Nonlinear Programming (NLP) for Stable, Robust and High Performance NMPC

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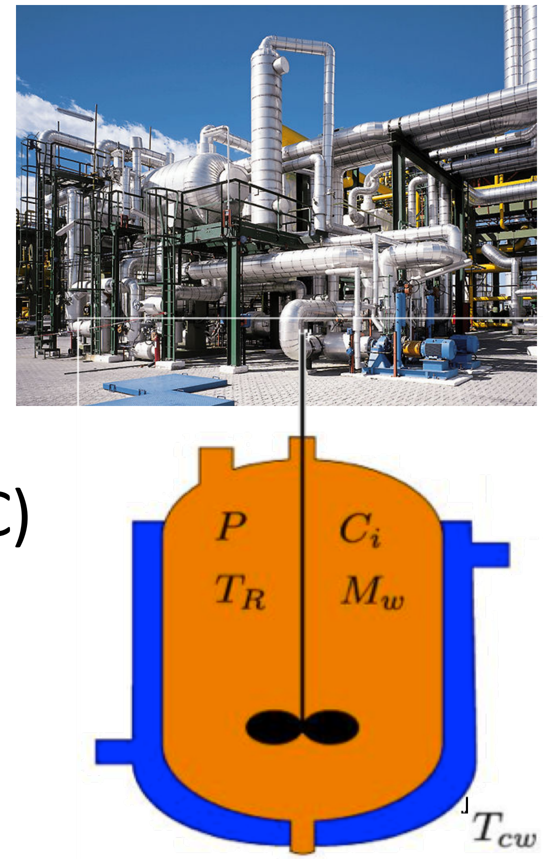
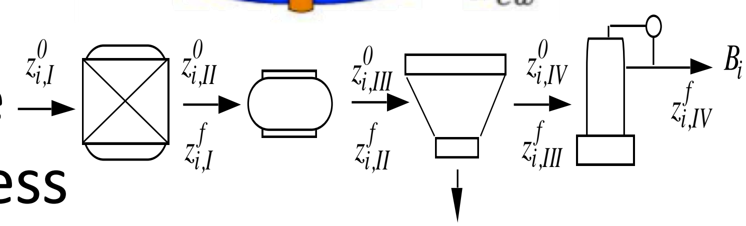
Model Predictive Control and Dynamic Optimization in Chemical Processes

- Model Predictive Control
  - MIMO with many states and constraints
  - Detailed nonlinear dynamic models
  - Slower dynamics (seconds, minutes)
  - Optimal reference trajectories



- Dynamic Real-time Optimization (eNMPC)
  - Moving Horizon Framework
  - Extensions of MPC
  - Stage costs may not be dissipative

- Can NLPs with first principle dynamics be solved on-line with stability and robustness guarantees?



NMPC Nominal Stability with Terminal Cost

(Rawlings and Mayne, 2009)

$$V(x(k)) = \min_{u} \sum_{l=0}^{N} \psi(z_l, v_l) + \Psi(z_N)$$

$$s.t. \quad \begin{aligned} & z_{l+1} = f(z_l, v_l)) \\ & z_0 = x(k),\ z_l \in X,\ v_l \in U, z_N \in X_f, \end{aligned}$$

$$==> u(k) = v_0,\ Set\ k = k+1$$

Assumptions:

- $f(x, u)$ is Lipschitz continuous *(will assume smooth)*
- There exists a local control law $u = \kappa_f(x)$ for all $x \in \mathcal{X}_\phi$ such that

  $\Psi(f(x, \kappa_f(x))) - \Psi(x) \leq -\psi(x, \kappa_f(x))$
- $\psi(x, u)$, $\Psi(x)$ satisfy $\alpha_p(|x|) \leq \psi(x, u)$, $\alpha_q(|x|) \leq \Psi(x)$ where $\alpha_p(\bullet)$, $\alpha_q(\bullet)$ are $\mathcal{K}$ functions.
- N sufficiently long, $\Psi(x)$ sufficiently large (Pannocchia, Rawlings, 2011) → no terminal constraints needed
- Stability Properties: Lyapunov Function based on Stage Costs

MFCQ for Reformulated NLP

$$
\begin{aligned}
V(x(k)) := \quad & \min_{v_l, z_l} \Psi(z_N) + \sum_{l=0}^{N-1} \psi(z_l, v_l) + \boxed{\sum_{l=0}^{N} \rho \xi_l} \\
s.t. \quad & z_{l+1} = f(z_l, v_l) \\
& z_0 = x(k) \\
& v_l \in U, l = 0, \ldots, N-1 \\
& \boxed{g(z_l) \le \xi_l, \xi_l \ge 0,, l = 0, \ldots, N} \\
& e = [1,1,1,1\ldots 1]^T
\end{aligned}
$$

- Choose ρ and a sufficiently large *N* to determine nominal stability
- Softened state constraints with exact penalties are enough to satisfy:
  - MFCQ: equalities have full rank Jacobian, ξ can always be increased
  - to satisfy: $\nabla g(z)^T d_z - d_\xi < 0$
  - CRCQ: inequalities are linear
  - GSSOSC: assume weights on quadratic stage costs are chosen sufficiently large
- **The solution and $V(x(k))$ are uniformly continuous**

KKT Properties and Constraint Qualifications for Sensitivity

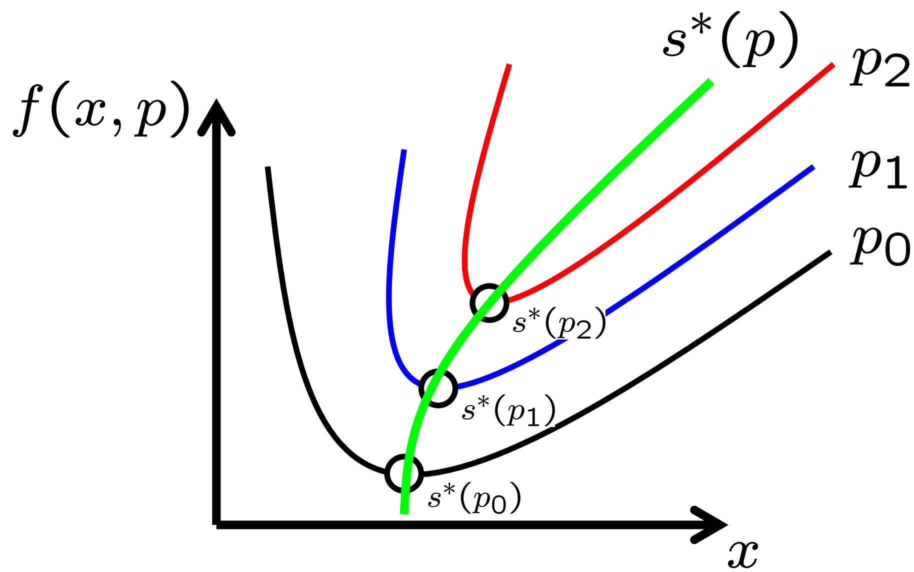
For $s^*(p) = (x^*, \lambda^*, \nu^*)$

- MFCQ, GSSOSC – uniform continuity of objective function and x^* with respect to p. (Kojima, 1985)
- MFCQ, GSSOSC, CRCQ → $(D_{\Delta p}\, x^*)$ - directional derivatives calculated with additional LP and QP steps (Ralph and Dempe, 1995)
- LICQ, SOSC, SC → (ds^*/dp) - derivatives can be calculated (Fiacco, 1983) ➔ KKT matrix is nonsingular

NLP Sensitivity

Parametric Program

$$\left.\begin{array}{ll} \min & f(x,p) \\ \text{s.t.} & c(x,p) = 0 \\ & x \geq 0 \end{array}\right\} \mathrm{P}(p)$$



Solution Triplet

$$s^*(p)^T = [x^{*T} \; \lambda^{*T} \; \nu^{*T}]$$

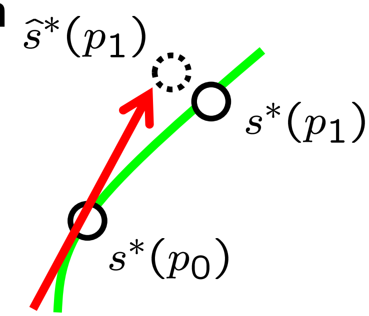
Optimality Conditions $\mathrm{P}(p)$

$$\begin{aligned} \nabla_x f(x,p) + \nabla_x c(x,p)\,\lambda - \nu &= 0 \\ c(x,p) &= 0 \\ XVe &= 0 \end{aligned}$$

NLP Sensitivity → Rely upon Existence and Differentiability of $s^*(p)$

→ **Main Idea: Obtain** $\left.\frac{\partial s}{\partial p}\right|_{p_0}$ **and find** $\widehat{s}^*(p_1)$ **by Taylor Series Expansion**

$$\widehat{s}^*(p_1) \approx s^*(p_0) + \left.\frac{\partial s}{\partial p}^T\right|_{p_0} (p_1 - p_0)$$



NLP Sensitivity with IPOPT
(Pirnay, Lopez Negrete, B., 2011)

Obtaining $\left.\frac{\partial s}{\partial p}\right|_{p_0}$

Optimality Conditions of $P(p)$: <u>Solved with Newton's Method in IPOPT</u>

$$\left.\begin{aligned} \nabla_x \mathcal{L} = \nabla_x f(x,p) + \nabla_x c(x,p)\,\lambda - \nu &= 0 \\ c(x,p) &= 0 \\ XVe &= 0 \end{aligned}\right\} \quad \mathbf{Q}(s,p) = 0$$

Apply Implicit Function Theorem to $\mathbf{Q}(s,p) = 0$ **around** $(p_0, s^*(p_0))$

$$\underbrace{\frac{\partial \mathbf{Q}(s^*(p_0),p_0)}{\partial s} \left.\frac{\partial s}{\partial p}\right|_{p_0} + \frac{\partial \mathbf{Q}(s^*(p_0),p_0)}{\partial p} = 0}$$

$$\begin{bmatrix} W(s^*(p_0)) & A(x^*(p_0)) & -I \\ A(x^*(p_0))^T & 0 & 0 \\ V^*(p_0) & 0 & X^*(p_0) \end{bmatrix} \begin{bmatrix} \frac{\partial x}{\partial p} \\ \frac{\partial \lambda}{\partial p} \\ \frac{\partial \nu}{\partial p} \end{bmatrix} + \begin{bmatrix} \nabla_{x,p} \mathcal{L}(s^*(p_0)) \\ \nabla_p\, c(x^*(p_0)) \\ 0 \end{bmatrix} = 0$$

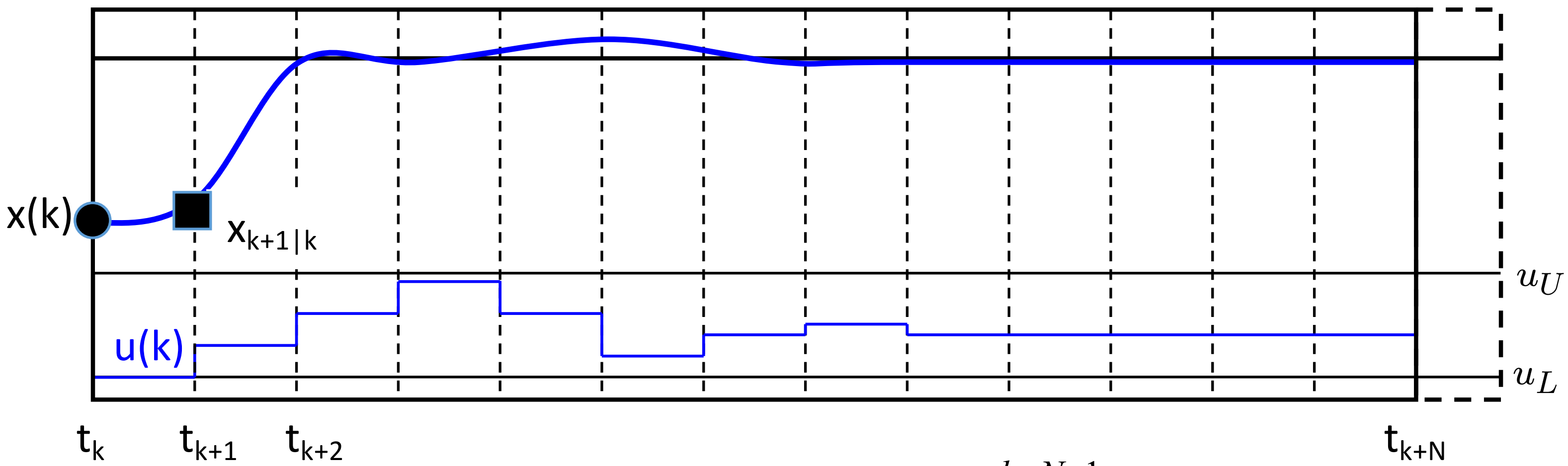
KKT Matrix IPOPT

$$\begin{bmatrix} W(x_k, \lambda_k) & A(x_k) & -I \\ A(x_k)^T & 0 & 0 \\ V_k & 0 & X_k \end{bmatrix}$$

- → **Already Factored from Newton Step in IPOPT**
- → **Sensitivity Calculation from Single Backsolve**
- → **Approximate Solution Retains Active Set**

Advanced Step Nonlinear MPC (Zavala, B., 2009)

Solve NLP in background (between steps, not on-line)
Update using sensitivity on-line



$$\min \quad V(x(k),\, u(k)) = F(x_{k+N|k}) + \sum_{l=k+1}^{k+N-1} \psi(x_{l|k}, v_{l|k})$$

$$s.t. \quad x_{k+1|k} = \boxed{f(x(k), u(k))}$$

$$x_{l+1|k} = f(x_{l|k}, v_{l|k}), \quad l = k+1, \ldots k+N\text{-}1$$

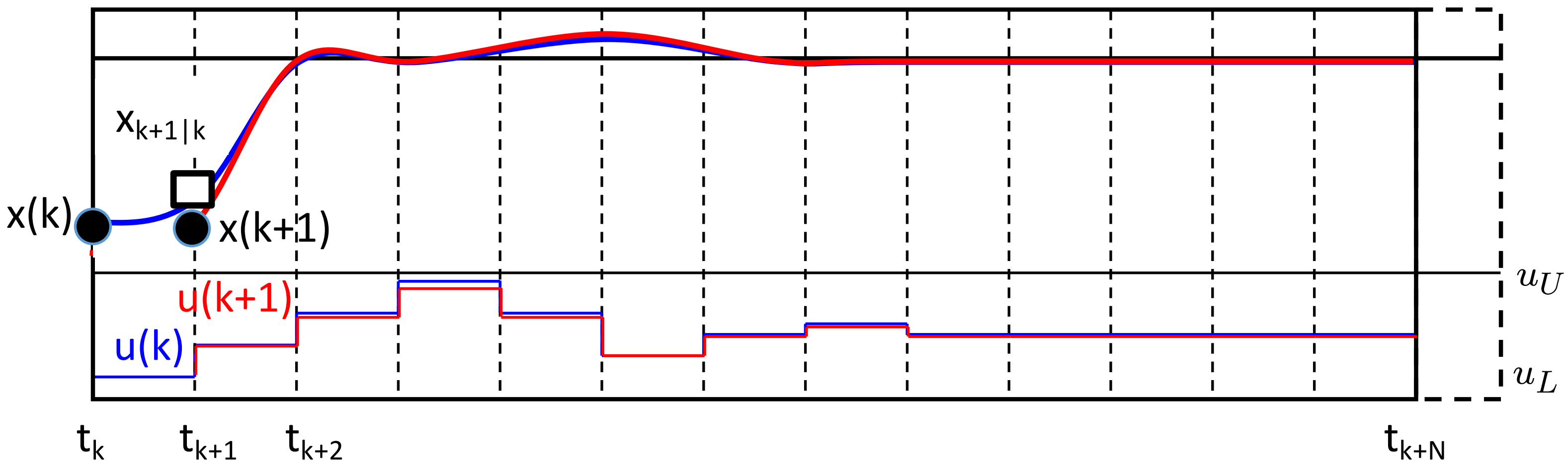
$$x_{l|k} \in X, \quad v_{l|k} \in U, \quad x_{k+N|k} \in X_f$$

Solve NLP(k) in background (between t_k and t_{k+1})

Offline Predictor

Advanced Step Nonlinear MPC (Zavala, B., 2009)

Solve NLP in background (between steps, not on-line)
Update using sensitivity on-line



$$\begin{bmatrix} W_k & A_k & -I \\ A_k^T & 0 & 0 \\ Z_k & 0 & X_k \end{bmatrix} \begin{bmatrix} \Delta \mathbf{x} \\ \Delta \lambda \\ \Delta z \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ x_{k+1|k} - x(k+1) \\ 0 \end{bmatrix}$$

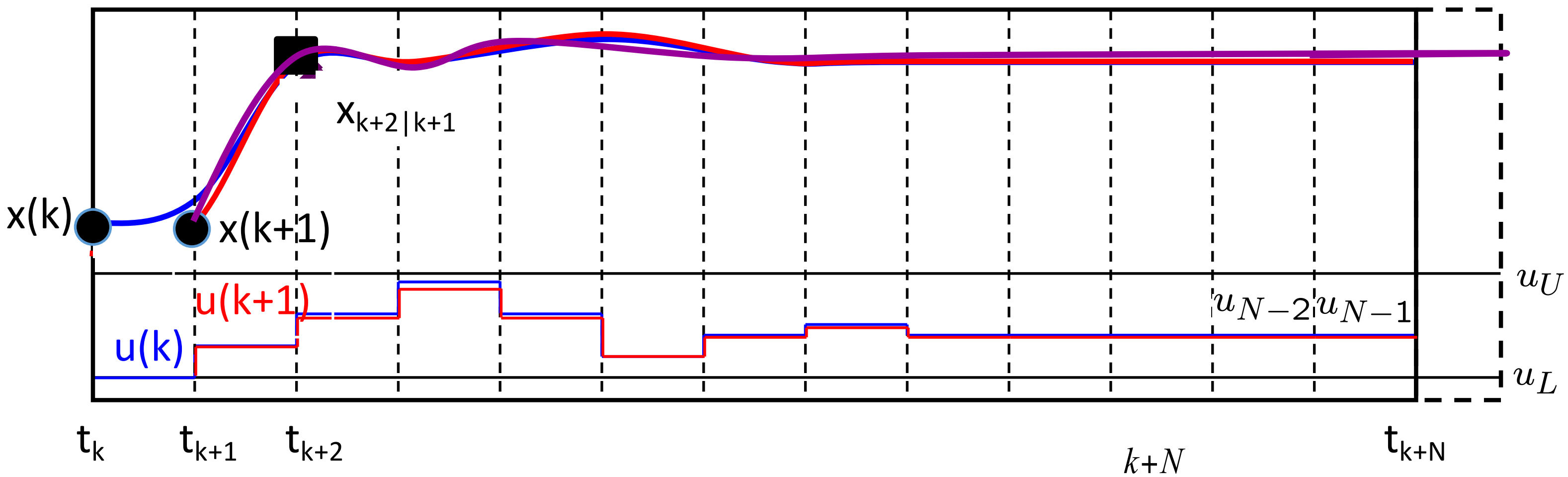
Solve NLP(k) in <u>background</u> (between t_k and t_{k+1})

Sensitivity to update problem <u>on-line</u> to get (u(k+1))

Online Corrector

Advanced Step Nonlinear MPC (Zavala, B., 2009)

Solve NLP in background (between steps, not on-line)
Update using sensitivity on-line



$$\min \quad V(x(k+1), u(k+1)) = F(x_{k+N+1|k+1}) + \sum_{l=k+2}^{k+N} \psi(x_{l|k+1}, v_{l|k+1})$$

$$s.t. \quad x_{k+2|k+1} = \boxed{f(x(k+1), u(k+1))}$$

$$x_{l+1|k+1} = f(x_{l|k}, v_{l|k}), \quad l = k+2, \ldots k+N$$

$$x_{l|k+1} \in X, \quad v_{l|k+1} \in U, \quad x_{k+N+1|k+1} \in X_f$$

Solve NLP(k) in background (between t_k and t_{k+1})

Sensitivity to update problem on-line to get (u(k+1))

Solve NLP(k+1) in background (between t_{k+1} and t_{k+2})

asNMPC: Concepts and Properties

- Interpretation: Fast linear MPC controller using *linearization of nonlinear model at previous step*.
- NLP solved between samples, “instantaneous” sensitivity update at sampling time
- On-line computation 2-3 orders of magnitude faster;

  ➔ Computational delay virtually eliminated
- Second order errors compared to ideal NMPC

  ➔ Nominal and ISS stability (Zavala, B., 2009)
- ISpS stability when coupled with embedded state estimators (Huang, Patwardhan, B., 2009a,b, 2010a-c, 2012)

Nonrobust NMPC Problem

Dynamic system:

$$f_1(x,u,w) = \frac{-(x_1^2+x_2^2)u + x_1}{1+(x_1^2+x_2^2)u^2 - 2x_1 u} + w_1$$

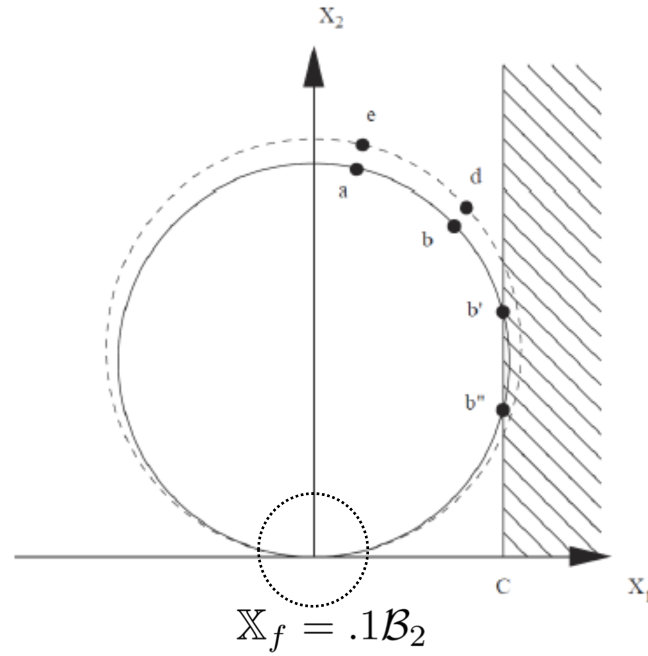
$$f_2(x,u,w) = \frac{x_2}{1+(x_1^2+x_2^2)u^2 - 2x_1 u} + w_2$$

Inequality Constraints:

$$\mathbb{X} = \{x : x \in \mathbb{R}^2, x_1 \leq .25\}$$

$$\mathbb{U} = [-1, 1]$$

$$\mathbb{X}_f = .1\mathcal{B}_2$$



Cost functions:

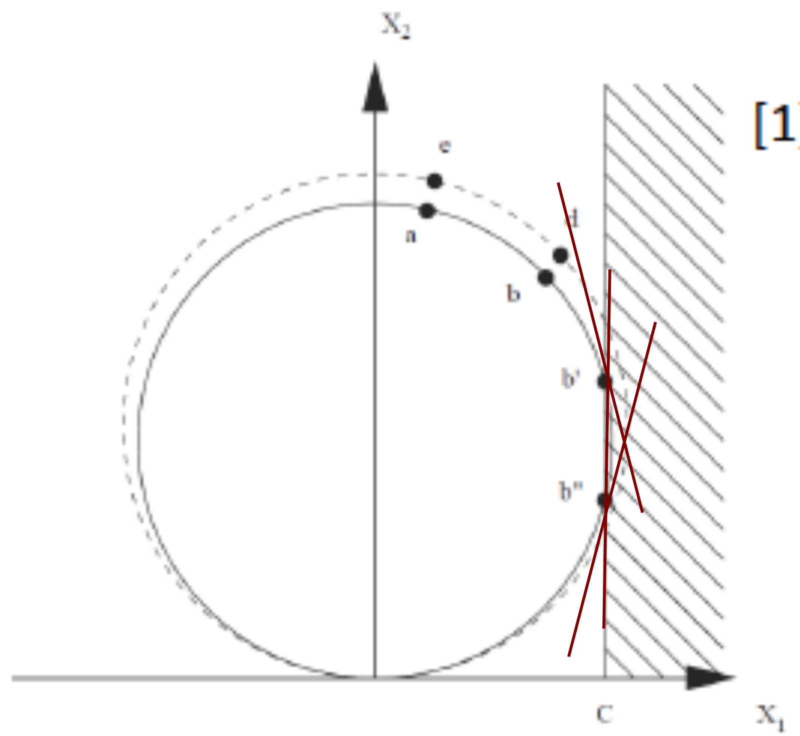
$$\Psi(x) = |x| \;\; cos^{-1}\delta \frac{(x_2-|x|)(-|x|)}{|x|\sqrt{x_1^2+(x_2-|x|)^2}}$$

$$\psi(x,u) = |x| \;\; cos^{-1}\delta \frac{x_1 f_1(x,-1) + (x_2-|x|)(f_2(x,-1)-|x|)}{\sqrt{x_1^2+(x_2-|x|)^2}\,\sqrt{f_1(x,-1)^2+(f_2(x,-1)-|x|)^2}}$$

Grimm, G., Messina, M. J., Tuna, S. and Teel, A. [2004], ‘Examples when nonlinear model predictive control is nonrobust’, *Automatica* ***40, 523–533.***

Source of Nonrobustness

- Nonrobustness is caused by the hard state constraint
- There exists a critical circle with radius $r_c = {}^{c}/_{\sqrt{1-c^2}}$ outside of which there exists no feasible c.w. solution
- Thus the value function of the NLP is discontinuous at this circle



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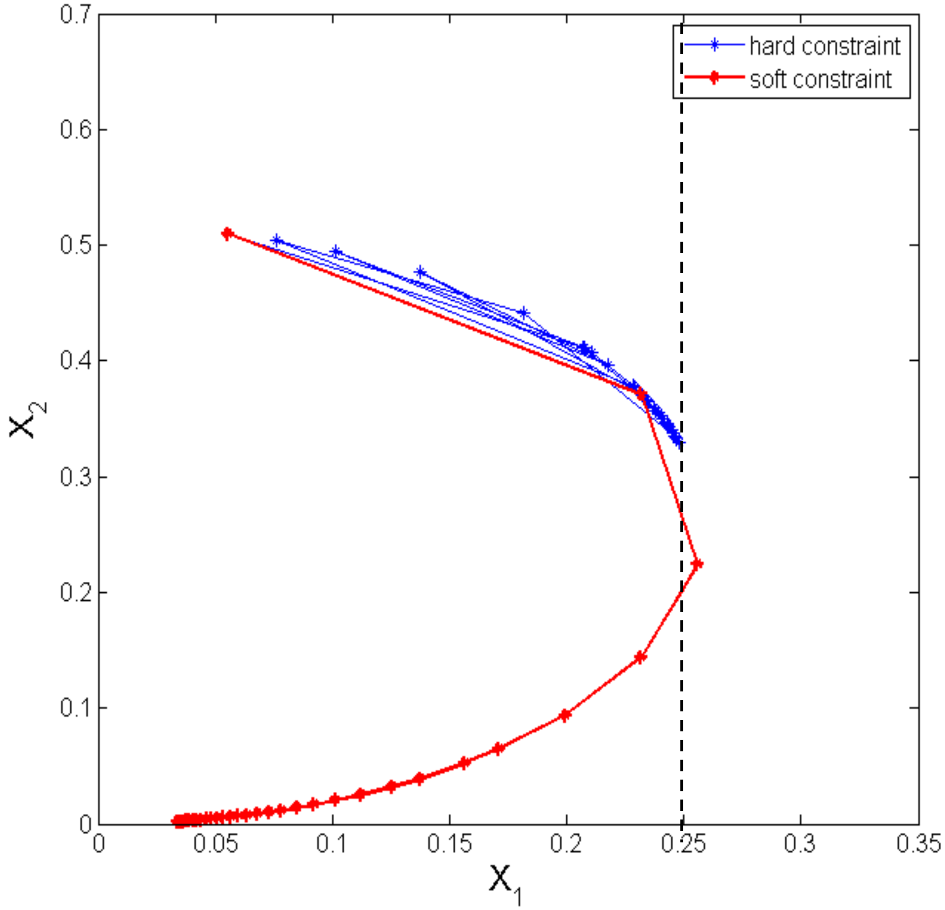
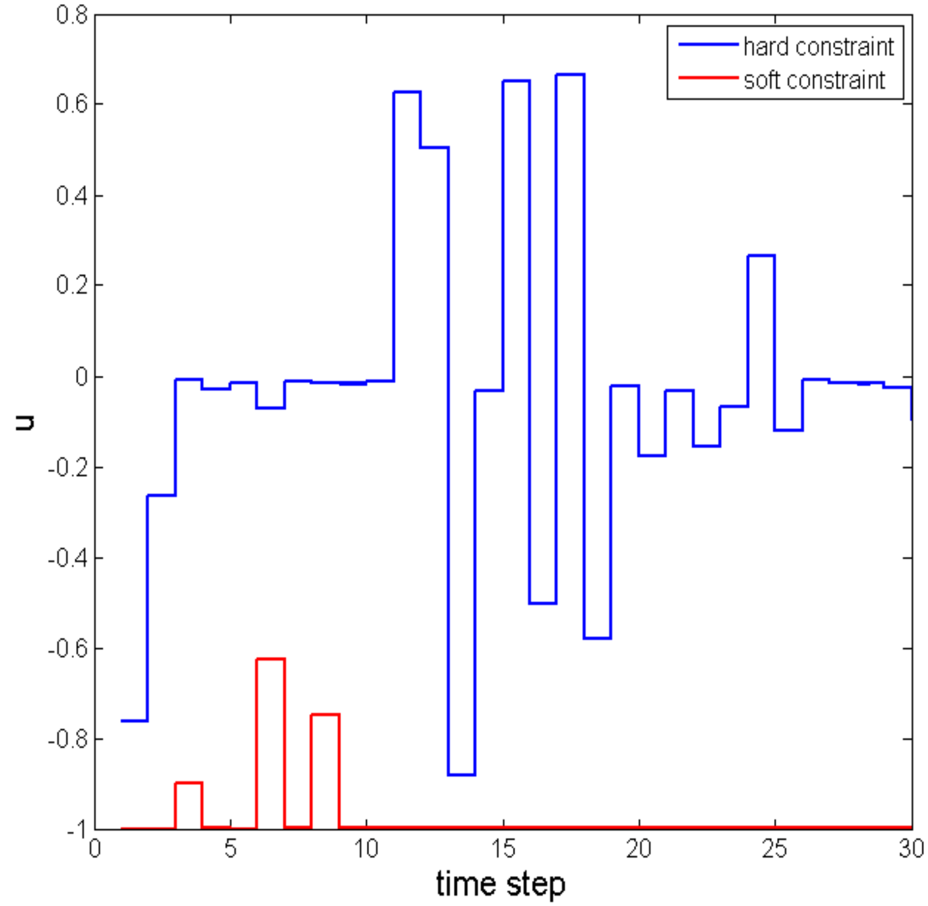
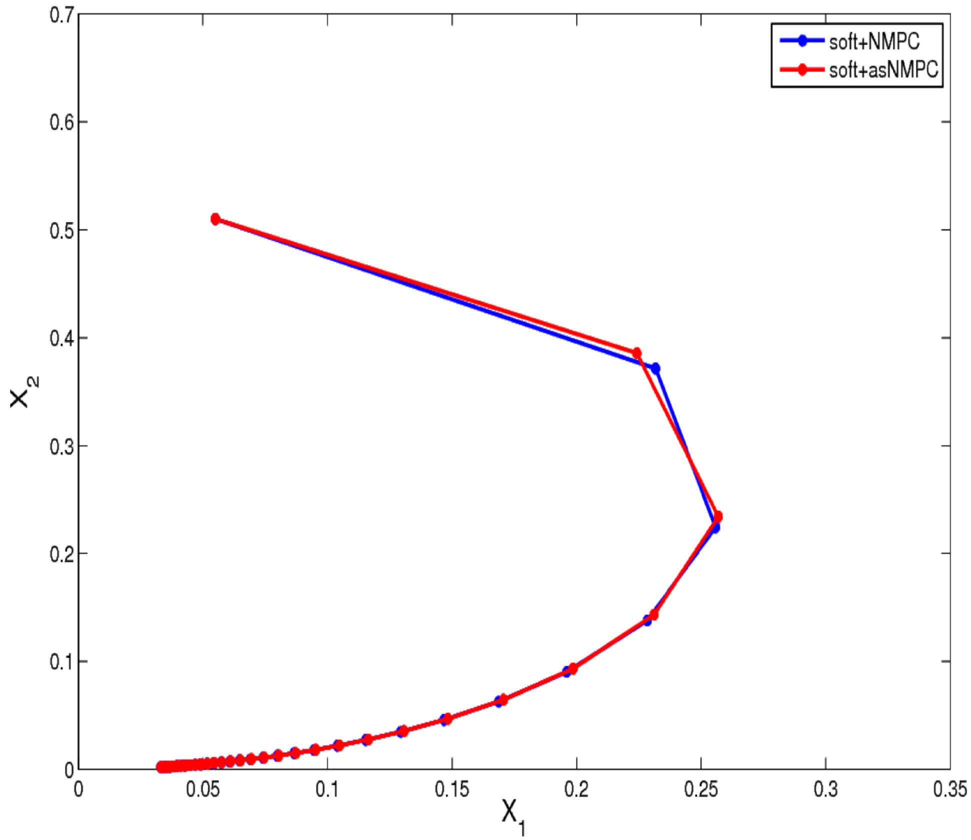
Impact of Reformulation
(ℓ_1 penalties and/or larger N)



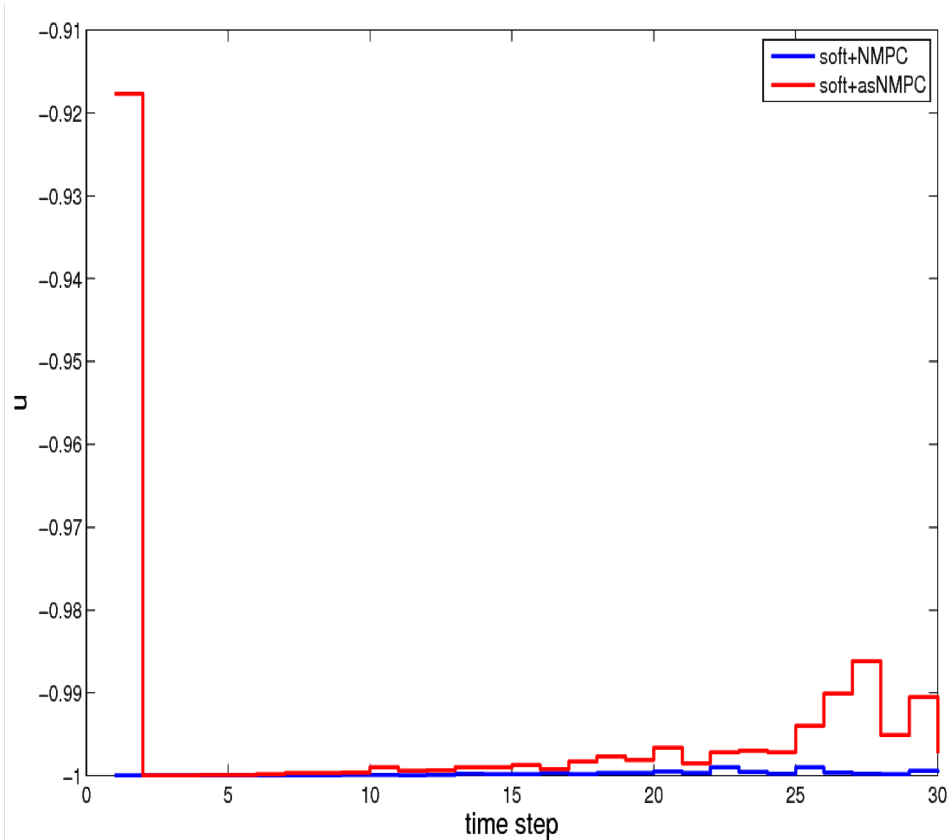


- Hard constraint $x_1 \leq c$ prevents trajectory from going beyond $x_1 = c$
- Soft constraint allows the trajectory to exceed $x_1 = c$ and converge
- MFCQ and GSSOSC satisfied

Ideal NMPC vs. asNMPC Results
(small amount of noise, σ = 0.05)



State Space Trajectory



Control Trajectory

Terminal Conditions

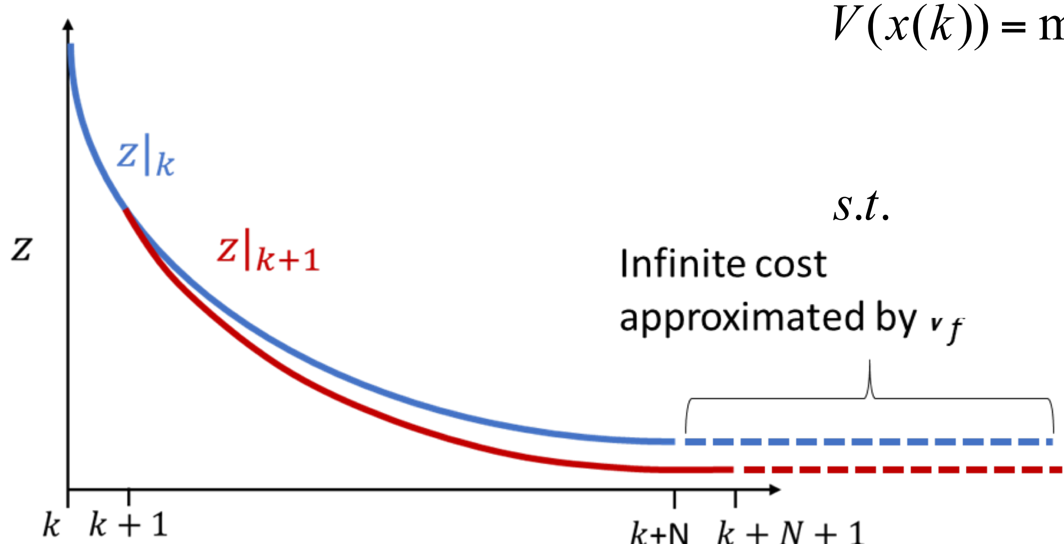
$$V(x(k)) = \min_{\boldsymbol{u}} \quad \sum_{l=0}^{N} \psi(z_l, v_l) + \boxed{\Psi(z_N)}$$

$$s.t. \quad \begin{aligned} & z_{l+1} = f(z_l, v_l)) \\ & z_0 = x(k),\; z_l \in X,\; v_l \in U, z_N \boxed{\in X_f,} \end{aligned}$$

$$==> u(k) = v_0,\; Set \;\; k = k+1$$

- Assume large, N --> ∞ ?
  - Hard to check, Can result in large NLPs
  - Typically ignored in real applications
- Endpoint constraint $z_N = 0$
  - Requires N controllability.
  - What is N? Potential difficulty in finding feasible solutions
- Terminal region $z_N \in \mathcal{X}_f$ and/or terminal cost $\Psi(z_N)$
  - Avoids problems of previous two methods
  - Selection of region or cost not obvious
  - Recursive feasibility requires $\Psi(x_{k+1}) - \Psi(x_k) \leq -\psi(x_k, u_k) \;\forall\; x_k \in \mathcal{X}_f$

Quasi-Infinite Horizon NMPC



$$V(x(k)) = \min_{u} \sum_{l=0}^{N} \psi(z_l, v_l) + \Psi(z_N)$$

$$s.t. \quad z_{l+1} = f(z_l, v_l))$$

$$z_0 = x(k),\ z_l \in X,\ v_l \in U, z_N \in X_f,$$

$$==> u(k) = v_0,\ Set\ k = k+1$$

- Framework for terminal conditions that bound infinite horizon problem
  - Accounts for what happens after predictive horizon
  - Robustness maintained as time proceeds
- Terminal cost Ψ represents infinite horizon controller stabilizing in $\mathcal{X}_f$
- Choose LQR (linear quadratic regulator) to bound V_∞ in the terminal region

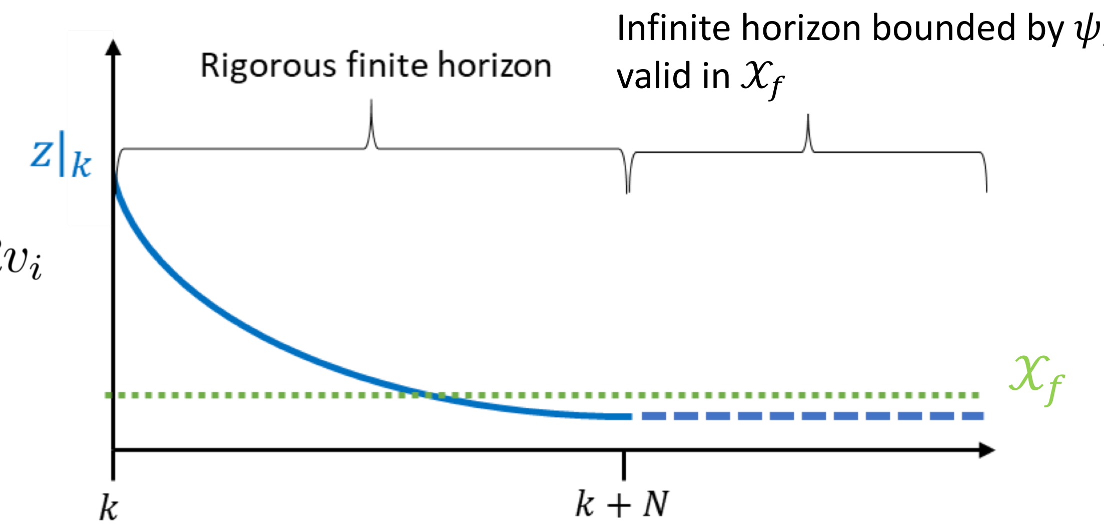
- H. Chen and F. Allgöwer. A quasi-infinite horizon nonlinear model predictive control scheme with guaranteed stability. *Automatica*, 34:1205–1218, 1998.
- C. Rajhans, S. Patwardhan, and H. Pillai. Two Alternate Approaches for Characterization of the Terminal Region for Continuous Time Quasi-Infinite Horizon NMPC. *Proceedings of the 12th IEEE International Conference on Control and Automation*, 98-103, 2016.
- M. Lazar, M. Tetteroo, IFAC Papers Online 51(20) (2018), pp. 141-146
- S. Lucia, P. Rumschinski, A. J. Krener, R. Findeisen, IFAC Papers Online 48(23) (2015), pp. 254-259

Terminal Cost Based on LQR

$$\psi(x) = x^T P x = \min_{z_i, v_i} \sum_{i=N}^{\infty} z_i^T Q z_i + v_i^T R v_i$$

$$s.t.\ \ z_{i+1} = A z_i + B v_i \ \ \forall\, i = 0 \ldots \infty$$

$$z_0 = x$$



- Terminal cost ψ becomes infinite horizon cost for linearized system
  - P found from Ricatti equation $P = A^T P A - A^T P B (R + B^T P B)^{-1} B^T P A + Q$
- **Computing terminal region $\mathcal{X}_f$ is equivalent to finding region where LQR stabilizes nonlinear system**
- Terminal region can be derived from Lyapunov function descent ==> provides bound on system nonlinearities
- To bound nonlinearities, method <u>must scale to many states</u>

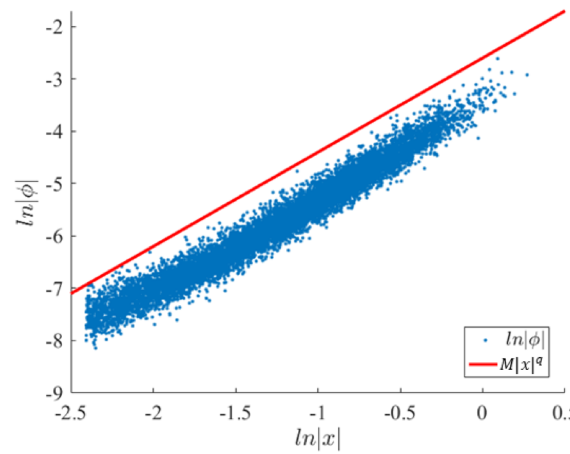
Bounding nonlinearities

- <u>Apply scalable method (off-line)</u>

$$|f(x, -Kx) - Ax + BKx| = |\phi(x)| \leq M|x|^q$$

  - Sample one-step CL-LQR simulations
  - Apply Taylor's Theorem
  - Simple and effective for large systems

Example of nonlinearity bound



- <u>$\mathcal{X}_f$ computed from M and q:</u>

- $\gamma_\psi > 0$ increases weight of LQR costs: $\widetilde{W} = (1 + \gamma_\psi)(Q + K^T RK)$
- Cost to go P and gain K satisfy the Lyapunov equation for the LQR
- Terminal region given by $\mathcal{X}_f = \{x \mid |x| \leq c_f\}$

$$c_f := \left(\frac{-\hat{\sigma}\Lambda_P + \sqrt{(\hat{\sigma}\Lambda_P)^2 + \lambda_{\Delta W}^{min}\Lambda_P}}{\Lambda_P M} \right)^{\frac{1}{q-1}}$$

- $\hat{\sigma}$ is the maximum singular value of $A_K = A - BK$ and $\Lambda_P = \dfrac{\lambda_{\widetilde{W}}^{max}}{1 - \hat{\sigma}^2}$
- Satisfies assumptions on terminal region/cost formulation for asymptotic stability

Robust QIH-NMPC Reformulation

$$V_N^r(x_k) = \min_{z_i, v_i} \sum_{i=0}^{N-1} \left(z_i^T Q z_i + v_i^T R v_i + \boxed{\rho \xi_i^T e} \right) + \boxed{z_N^T P z_N} + \boxed{\rho \xi_N^T e}$$

$$s.t. \;\; z_{i+1} = f(z_i, v_i) \;\; \forall\, i = 0 \ldots N-1$$

$$z_0 = x_k$$

$$h(z_i) \boxed{\le \xi_i, \xi_i \ge 0} \;\forall\, 0 = 1 \ldots N-1$$

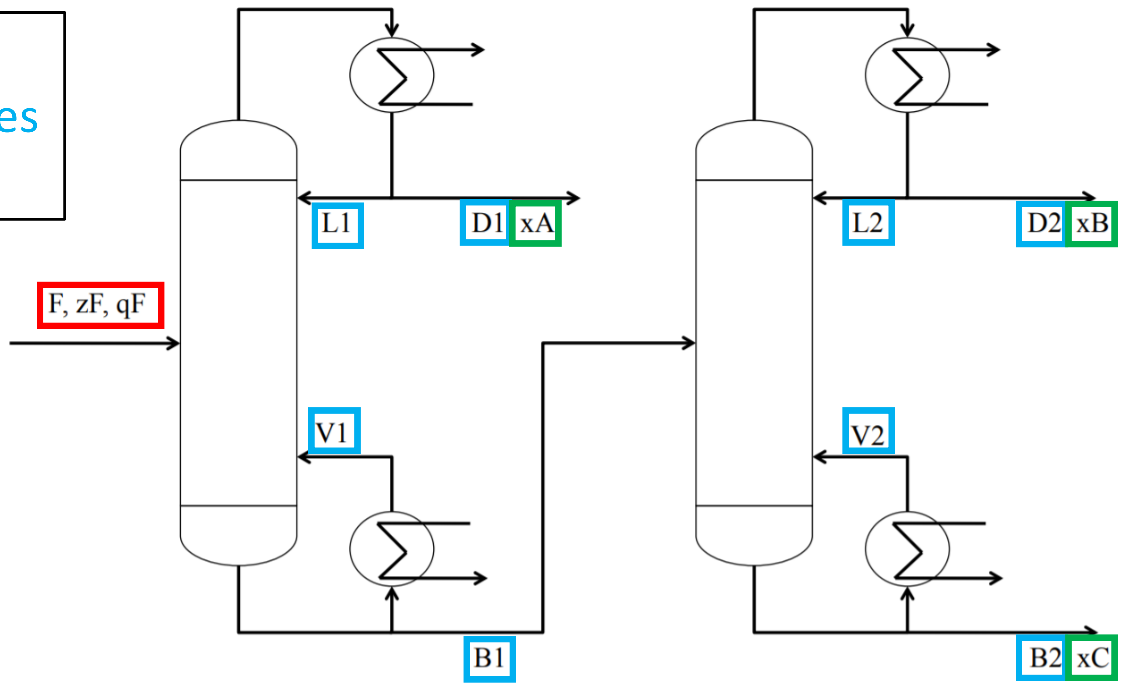
$$v_i \in \mathbb{U} \;\; \forall\, i = 0 \ldots N-1$$

$$\boxed{|z_N| \le c_f} + \boxed{\xi_N, \xi_N \ge 0}$$

- Penalty weight ρ chosen sufficiently large to inherit nominal stability
- Formulation satisfies MFCQ (feasible search direction exists)
- GSSOSC holds if Q, R large enough
- Thus V_N^r is uniformly continuous ==> Input-to-state stability (ISS) holds

Distillation Example

Inputs
Manipulated variables
Controlled variables



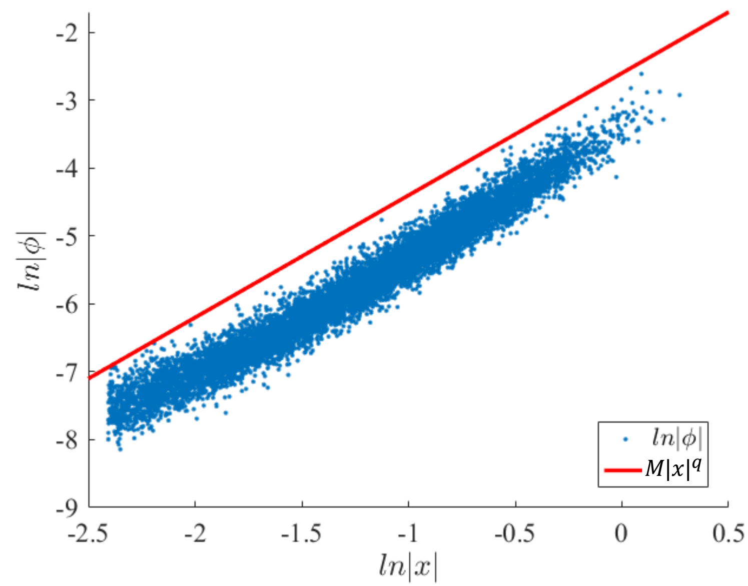
- Separation of three component mixture A,B,C - maintain product purities
- 246 States - tray holdups and compositions at 41 trays in each column
- 8 Controls - reflux, boilup, distillate flow, bottoms flow
- ~ 10000 variables, 200 dofs

R.B. Leer. Self-optimizing control structures for active constraint regions of a sequence of distillation columns. Master's thesis, Norwegian University of Science and Technology, 2012.

Terminal Regions for Distillation

- Nonlinearity bound set with q= 1.8 and $M = 0.0743$
- 10,000 one-step simulations under LQR (10 CPU min done offline)
- State and control constraints imposed independently
- Terminal region size and NMPC performance compared as function of terminal cost weight γ_ψ

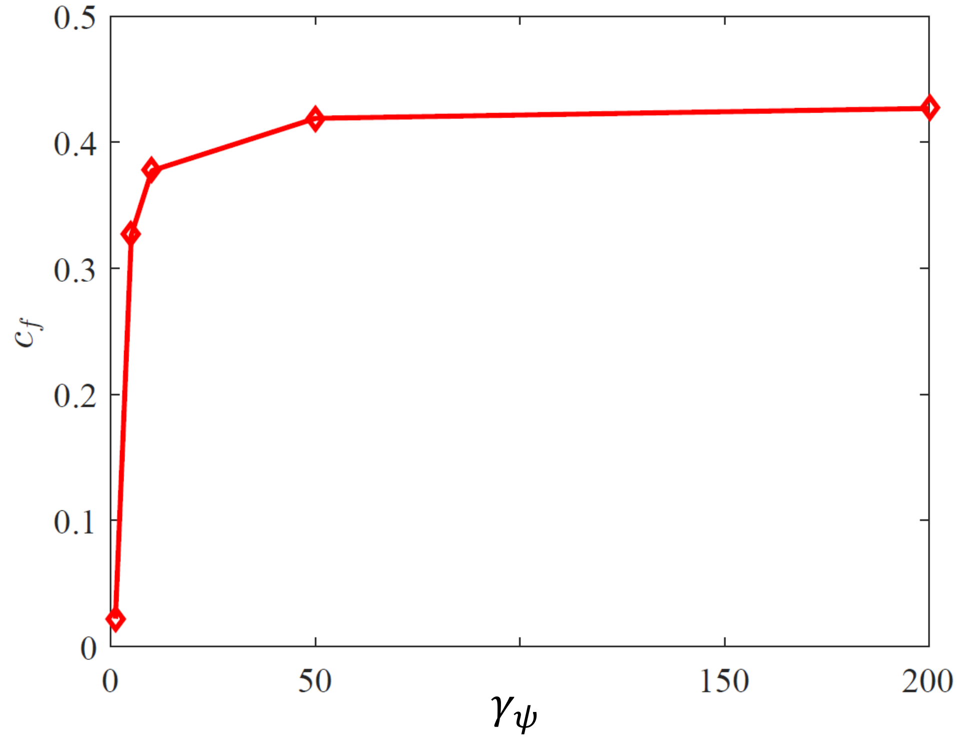
Nonlinearity bound



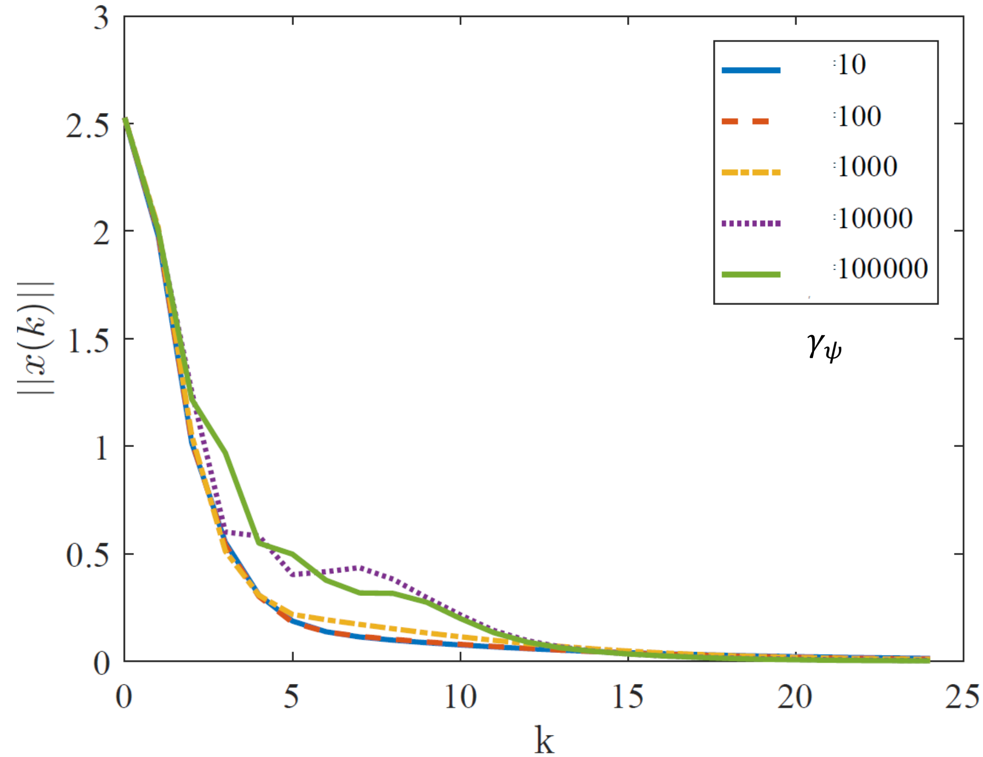
$$|f(x, -Kx) - Ax + BKx| = |\phi(x)| \leq M|x|^q$$

Distillation Results

Terminal region size



NMPC performance with N=10



- Terminal region given by $\mathcal{X}_f = \{z_N \mid |z_N| \leq c_f\}$
- LQR cost given by $(1 + \gamma_\psi)(Q + K^T R\ K)$
- Terminal region size increases with terminal cost weight, but plateaus
- Tracking performance degrades with large terminal cost weight
- ==> choose moderate weights

Adaptive Horizon NMPC: How long is N?

$$V_N(x_k) = \min_{z_i, v_i} \sum_{i=0}^{N-1} \left(z_i^T Q z_i + v_i^T R v_i \right) + z_N^T P z_N$$

$$s.t. \;\; z_{i+1} = f(z_i, v_i) \;\; \forall\, i = 0 \ldots N-1$$

$$z_0 = x_k$$

$$z_i \in \mathbb{X} \;\; \forall\, i = 0 \ldots N-1$$

$$v_i \in \mathbb{U} \;\; \forall\, i = 0 \ldots N-1$$

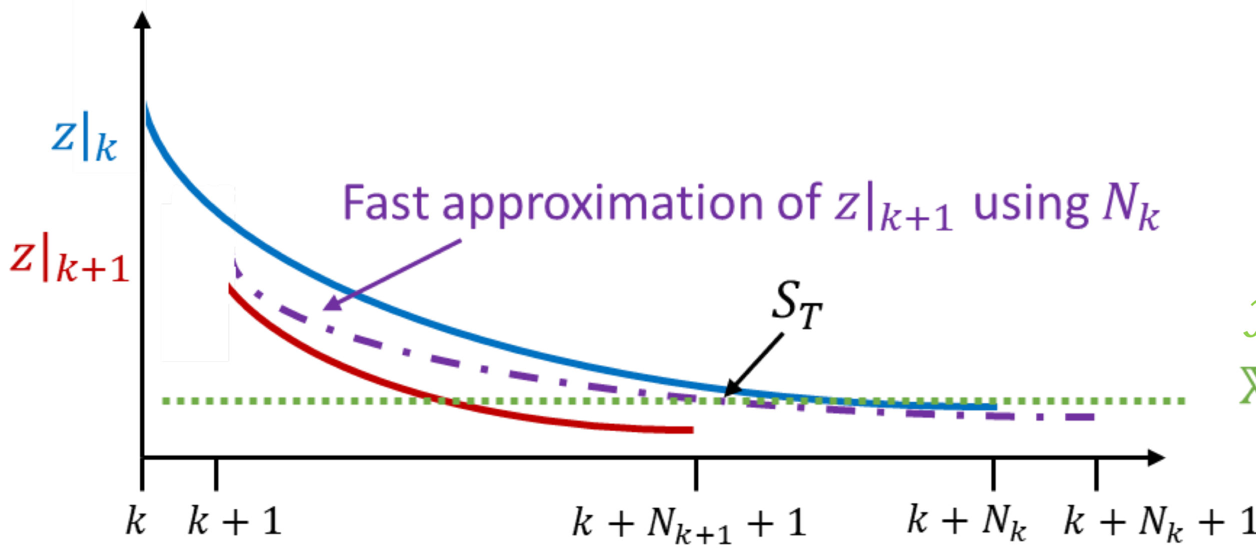
$$|z_N| \leq c_f$$

- Horizon length N balances computation and robustness
  - N too long: long solve times ⇒ delayed control actions
  - N too short: limited robustness ⇒ unstable or infeasible
  - Practical applications must use conservatively long horizon lengths
- Magnitude of trade-off changes with system state
  - Longer horizon necessary further from steady state
- Adaptively choose N in real time

- D. W. Griffith, S. C. Patwardhan and LTB, Journal of Process Control , 70, pp. 109{122 (2018)
- A. J. Krener, ArXiv:1062.08619 (2016)

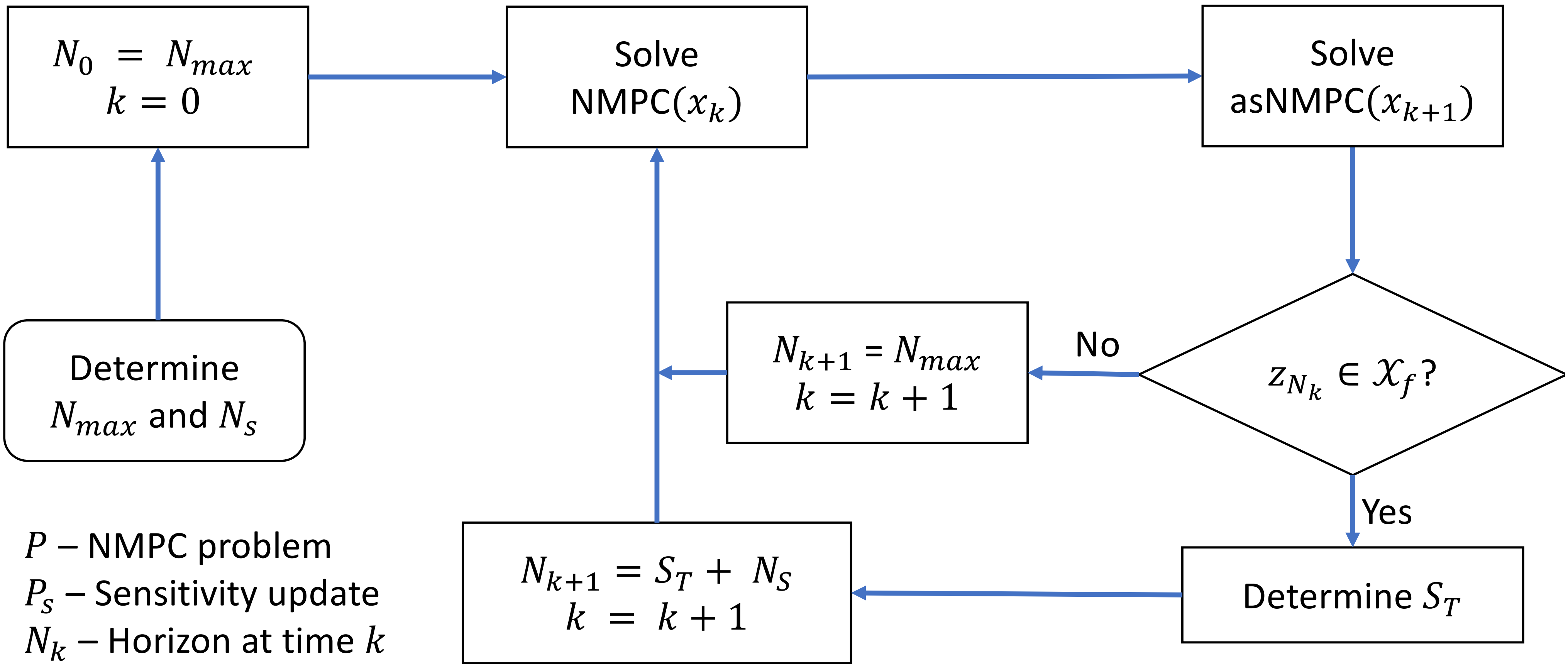
Horizon Length Selection

- NLP Sensitivity provides fast updates for perturbations to parametric NLPs
- Fast (~1 sec) estimates to NMPC problems obtained online by treating initial condition as parameter, as in asNMPC
- Approach: use sIpopt to predict the time step S_T at which the NMPC solution will reach $\mathcal{X}_f$



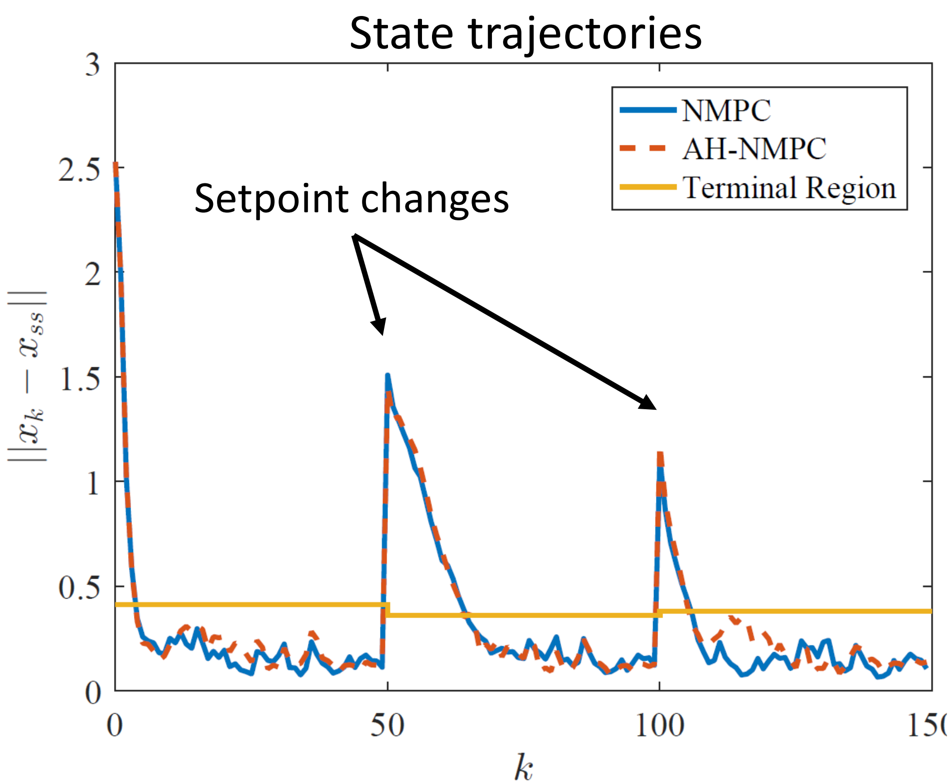
- Pirnay, H., Lopez-Negrete, R., LTB. (2012). Optimal sensitivity based on Ipopt. Math. Prog. Comp., 4, 307-331.
- Zavala, V.M., LTB. (2009). The advanced-step NMPC controller: Optimality, stability, and robustness. Automatica, 45, 86-93.

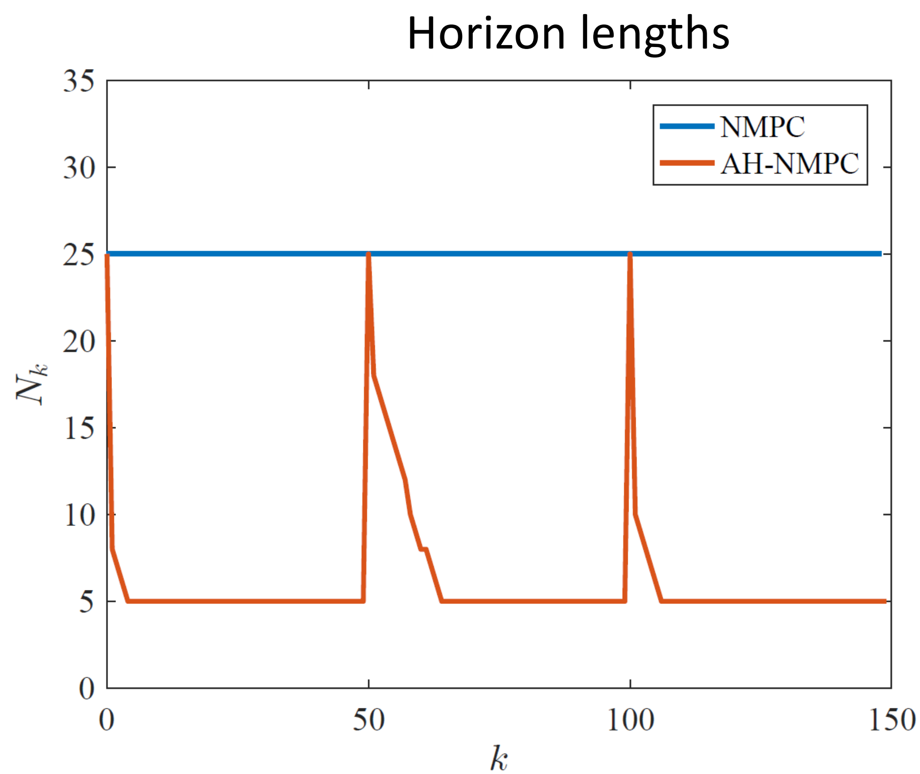
Adaptive Horizon Algorithm



P – NMPC problem
P_S – Sensitivity update
N_k – Horizon at time k
N_{max} – guarantees feasibility
N_S – safety factor
S_T – Time step at which the solution reaches the terminal region
x_k – system state at time k
z_{N_k}– sensitivity prediction of terminal state at time k

Distillation Example with Noise

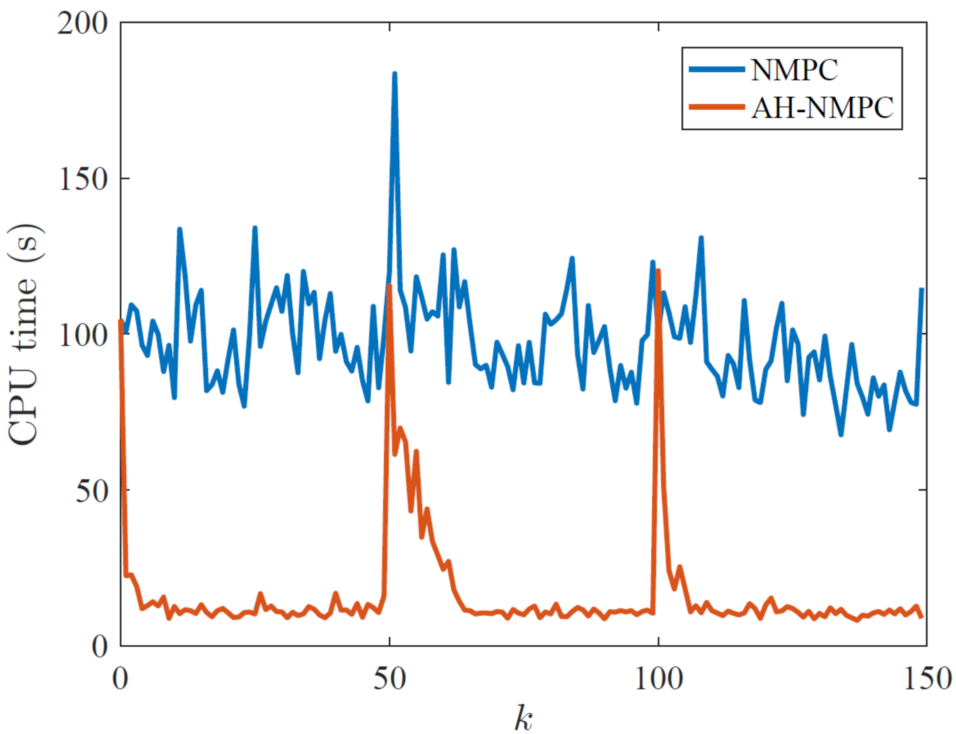




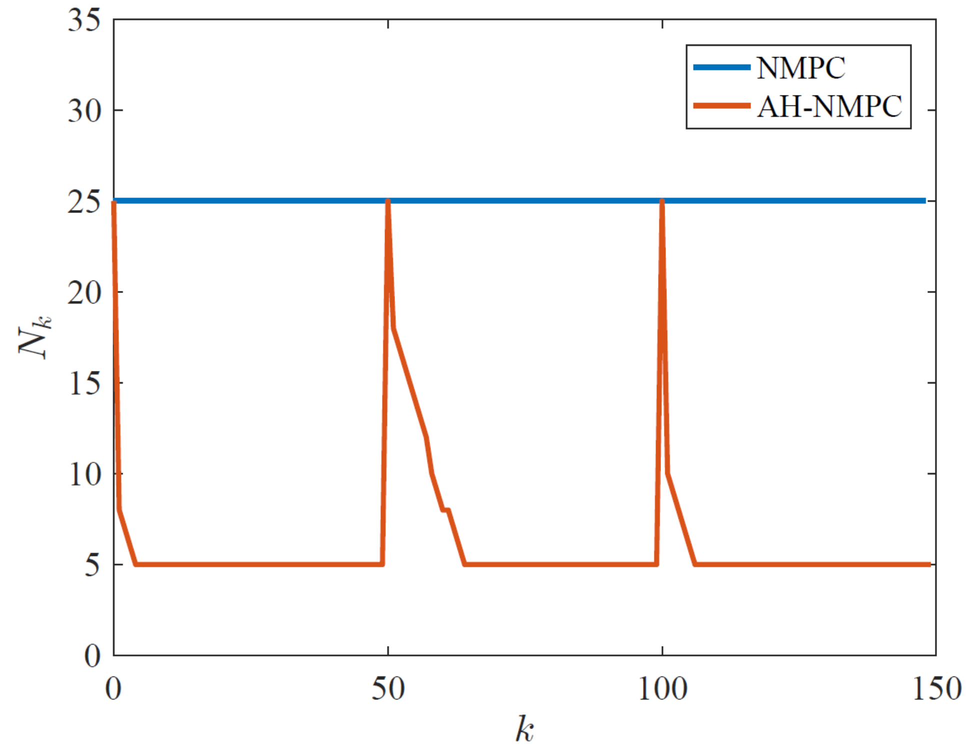
- Noise: 10% variance in feed flow and composition
- Horizon selection is robust, all problems are feasible
- Tracking performances similar to noise-free case
- AH-NMPC contracts horizon as the setpoint is approach, similar performance as with N = 25
- Feasible horizons chosen despite noise

Solve Times with Noise

Solve times



Horizon lengths



<u>(Off-line) Solve Times</u>

- NMPC average solve time <u>97 CPU s</u>
- AH-NMPC, average <u>17 CPU s</u>
- Application of asNMPC --> additional on-line computational savings

Conclusions

- Robust NLP reformulations
  - Uniform continuity/sensitivity of NLP guaranteed by KKT conditions and CQs (SSOSC, MFCQ, LICQ)
  - Soft output constraints lead to robustly stable NMPC
- Advanced Step NMPC
  - Fast off-line solutions
  - Virtually no on-line computation
  - Leads to ISS Stability
- Terminal conditions for large scale systems
  - Allows for reachability analysis and ultimately shorter horizons
  - Based on LQR control in $\mathcal{X}_f$, and applying Taylor expansions
  - Easily embedded in NMPC formulation
- Adaptive Horizon NMPC
  - Faster solve times via horizon length adaption utilizing sIPOPT
  - Robustness stability properties retained

Current and Future Work

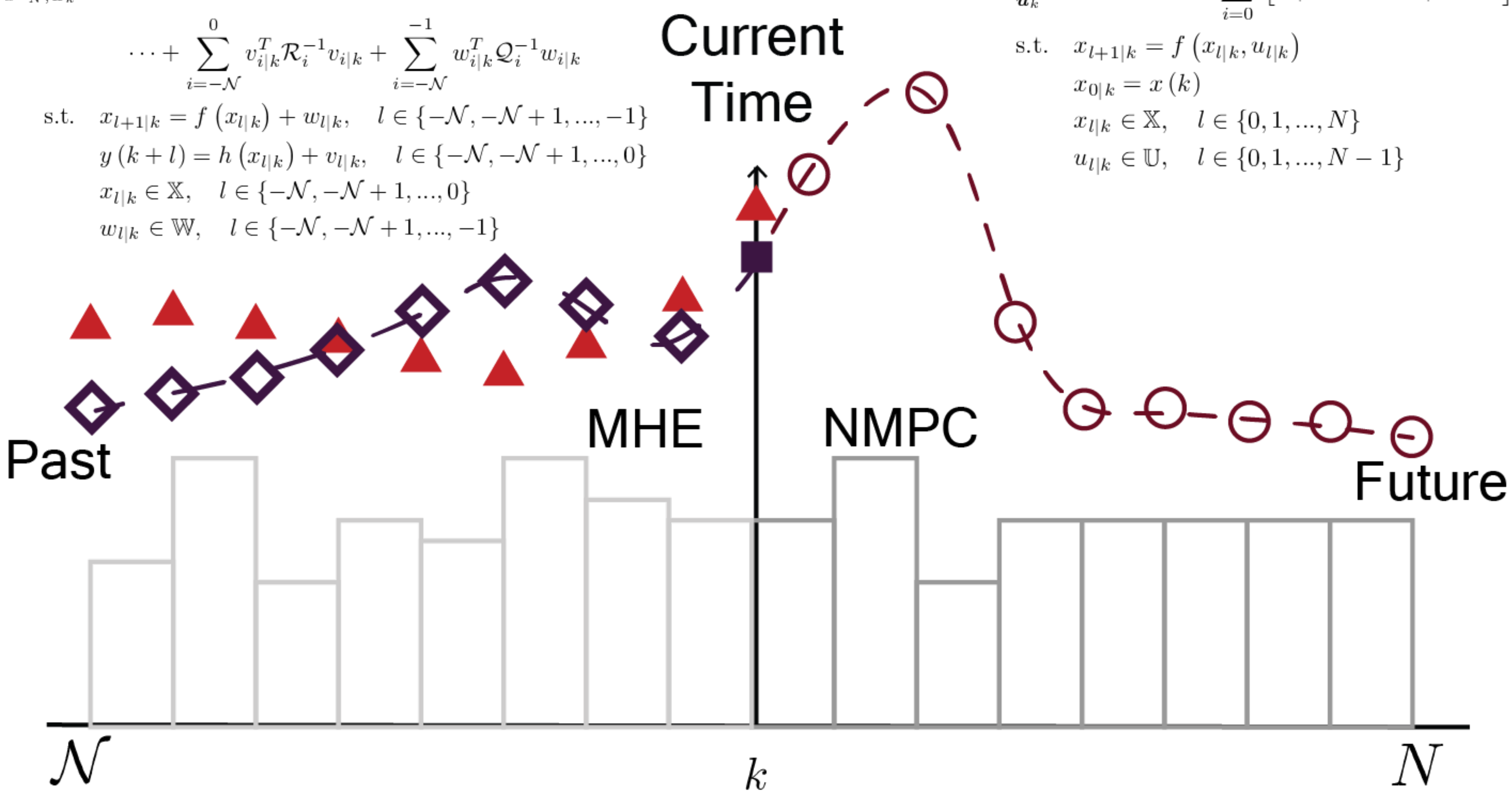
- Advanced Step Moving Horizon State Estimation

$$\mathcal{M}\left(\Pi_{-N|k-1}, \hat{x}_{-N|k-1}, y(k), \ldots, y(k-\mathcal{N})\right):$$

$$\begin{aligned}
\min_{x_{-N}, \boldsymbol{w}_k} \quad & \Phi_{-N}\left(x_{-\mathcal{N}|k}, \hat{x}_{-\mathcal{N}|k-1}, \Pi_{-\mathcal{N}|k-1}\right) + \cdots \\
& \cdots + \sum_{i=-\mathcal{N}}^{0} v_{i|k}^T \mathcal{R}_i^{-1} v_{i|k} + \sum_{i=-\mathcal{N}}^{-1} w_{i|k}^T \mathcal{Q}_i^{-1} w_{i|k} \\
\text{s.t.} \quad & x_{l+1|k} = f\left(x_{l|k}\right) + w_{l|k}, \quad l \in \{-\mathcal{N}, -\mathcal{N}+1, \ldots, -1\} \\
& y(k+l) = h\left(x_{l|k}\right) + v_{l|k}, \quad l \in \{-\mathcal{N}, -\mathcal{N}+1, \ldots, 0\} \\
& x_{l|k} \in \mathbb{X}, \quad l \in \{-\mathcal{N}, -\mathcal{N}+1, \ldots, 0\} \\
& w_{l|k} \in \mathbb{W}, \quad l \in \{-\mathcal{N}, -\mathcal{N}+1, \ldots, -1\}
\end{aligned}$$

$$\mathcal{P}\left(x(k)\right):$$

$$\begin{aligned}
\min_{\boldsymbol{u}_k} \quad & \varphi_N\left(x_{N|k}\right) + \sum_{i=0}^{N-1} \left[x_{i|k}^T Q x_{i|k} + u_{i|k}^T R u_{i|k}\right] \\
\text{s.t.} \quad & x_{l+1|k} = f\left(x_{l|k}, u_{l|k}\right) \\
& x_{0|k} = x(k) \\
& x_{l|k} \in \mathbb{X}, \quad l \in \{0, 1, \ldots, N\} \\
& u_{l|k} \in \mathbb{U}, \quad l \in \{0, 1, \ldots, N-1\}
\end{aligned}$$



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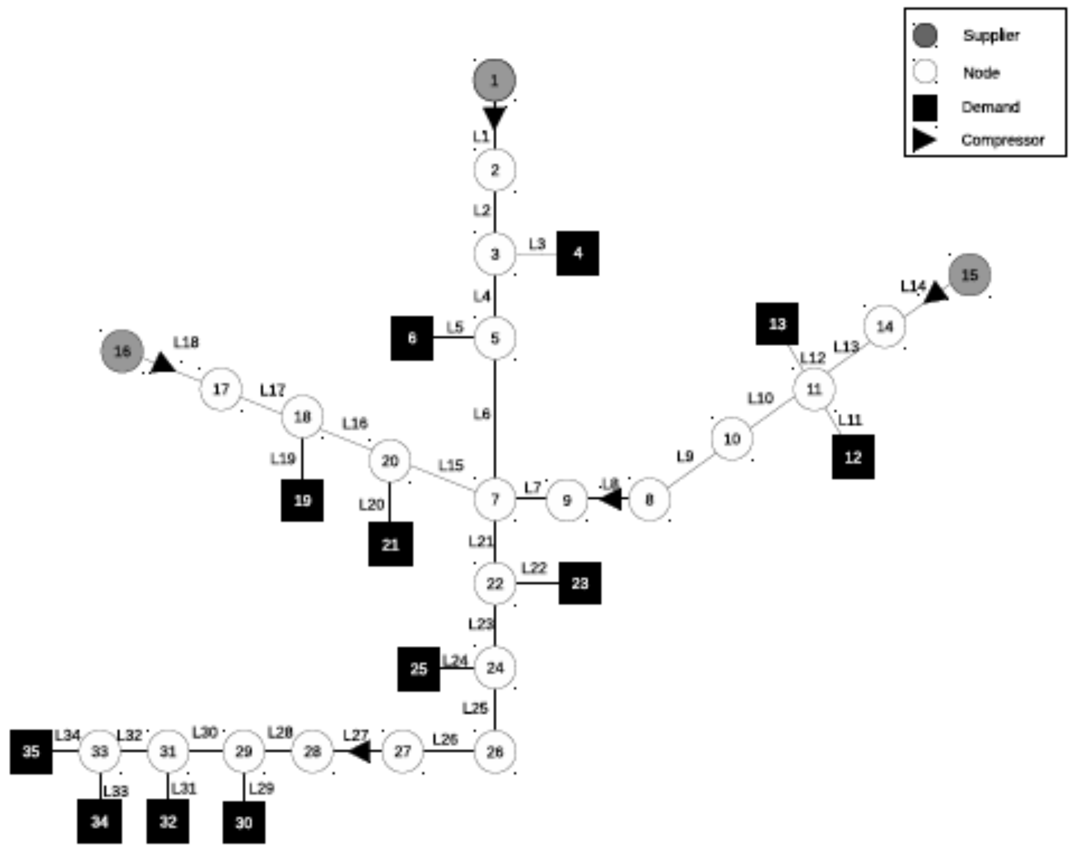
Current and Future Work

- Advanced Step Moving Horizon State Estimation
- Embedded discrete decisions for nonsmooth dynamics



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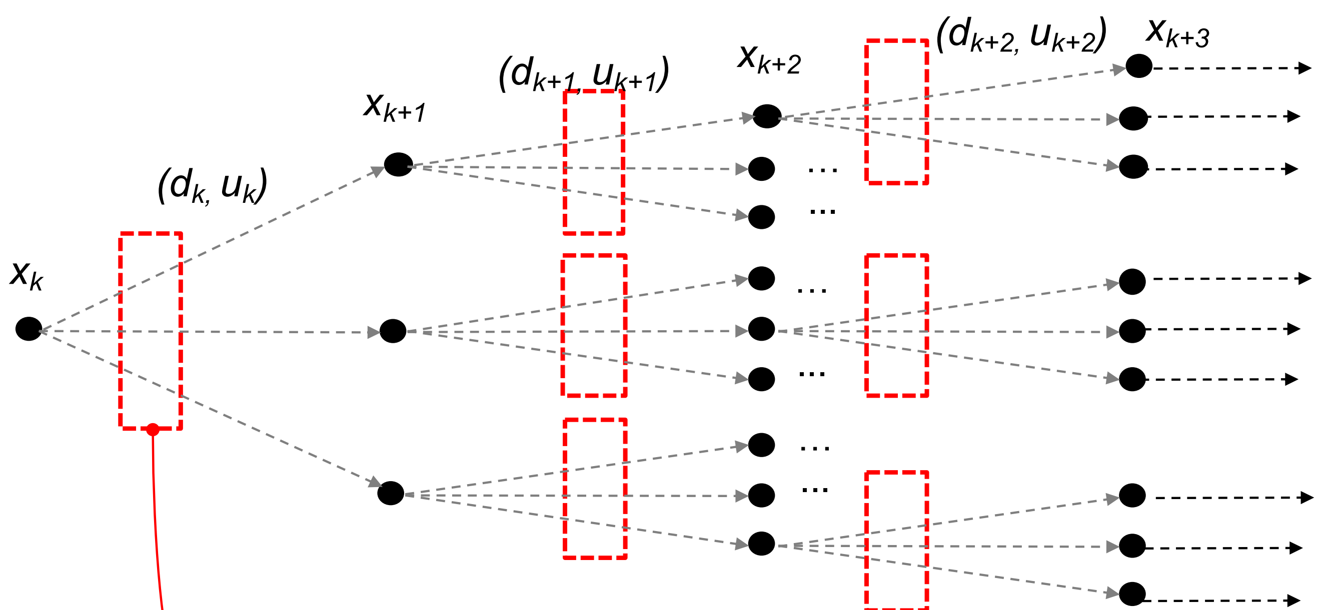
- Kai Liu, Saif R. Kazi, LTB, Bingjian Zhang,, Qinglin Chen, "Dynamic optimization for gas blending in pipeline networks with gas interchangeability control," submitted for publication (2019)

Current and Future Work

- Advanced Step Moving Horizon State Estimation
- Embedded discrete decisions for nonsmooth dynamics
- Multi-stage Stochastic formulations for NMPC with uncertainties and recourse variables

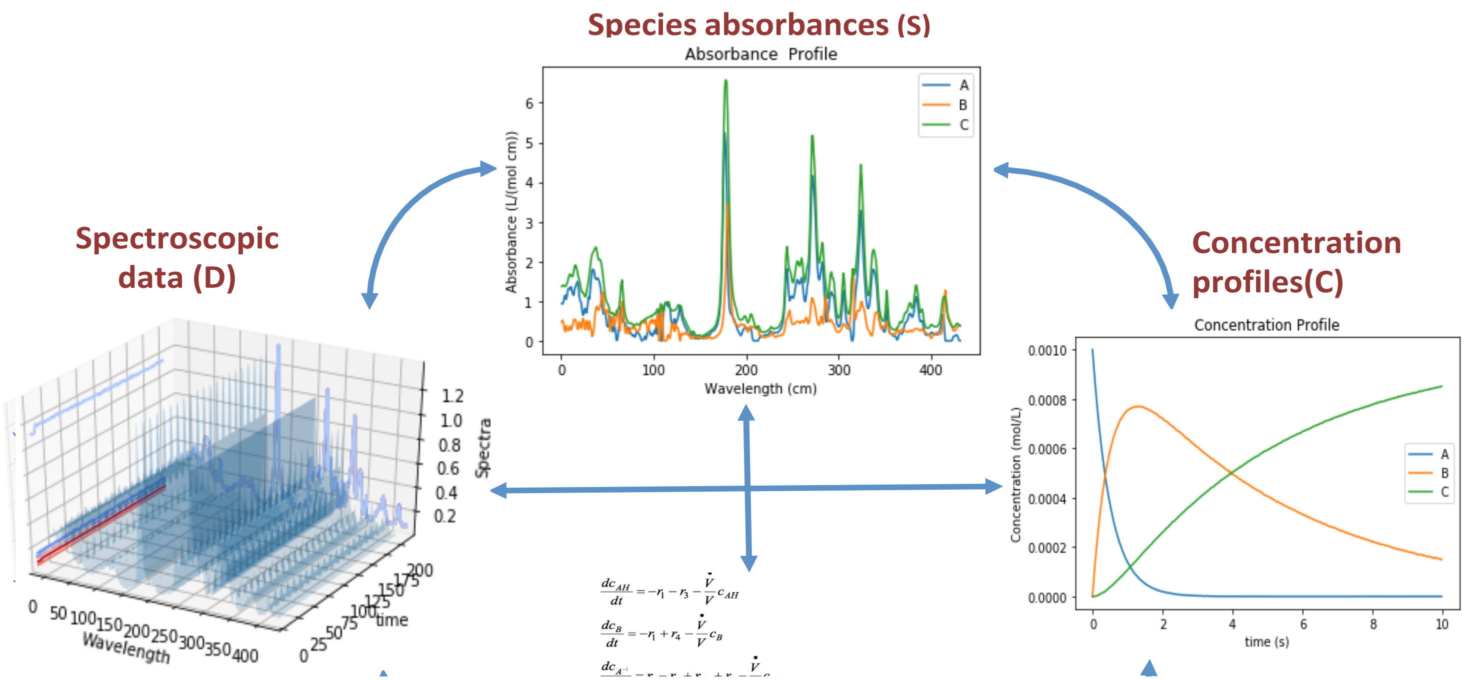
Scenario branching: effect of uncertainty while optimizing control input



Non-anticipativity: control inputs from same node set equal until uncertainty is realized

F. Holtorf, A. Mitsos, LTB, "Multistage NMPC with on-line generated scenario trees: Application to a semi-batch polymerization process," Journal of Process Control , 80, pp. 167-179 (2019)

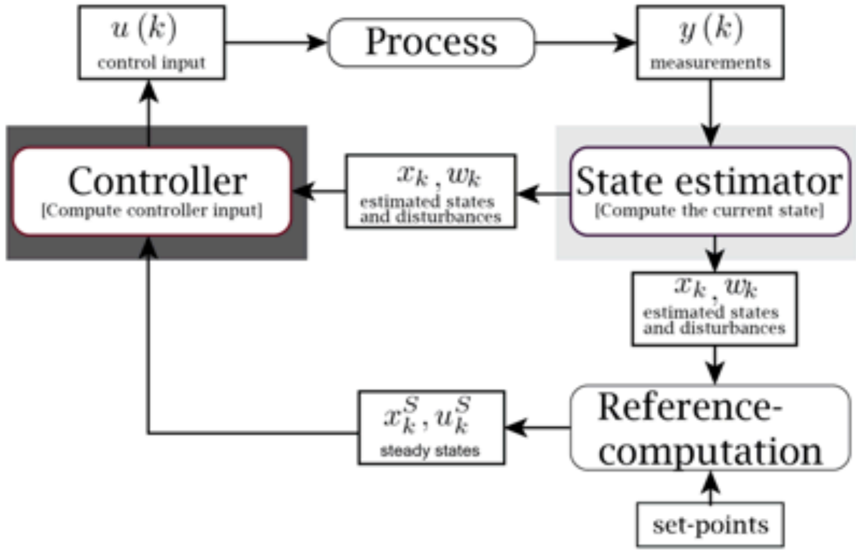
Current and Future Work

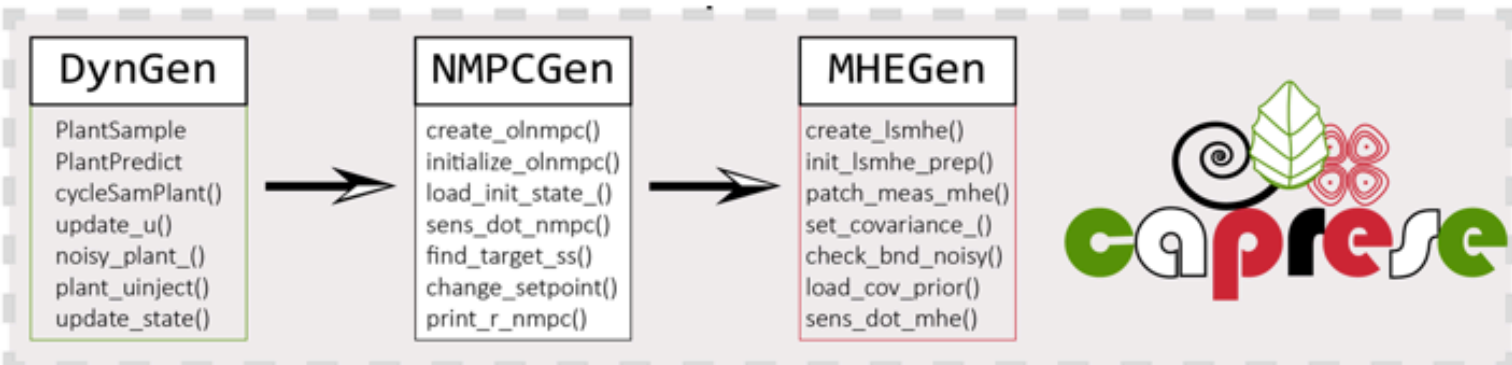


- Larger, more challenging applications
  - Big data in MHE (spectral measurements)
  - PDEs as process models
  - Exploit multiple time scales (ODEs --> DAEs)

M. Short, C. Schenk, D. Thierry, J. S. Rodriguez, LTB, S. Garcia-Munoz, “KIPET, An Open Source Kinetic Parameter Estimation Toolkit," Proc. 9 Intl Conference on Foundations of Computer-Aided Process Design, 293-302, (2019)

Current and Future Work





- <u>CAPRESE: </u>Python/Pyomo framework for asNMPC/asMHE and Sufficient horizon lengths found via sIPOPT

D. M. Thierry, LTB, "Dynamic Real-time Optimization for a CO2 Capture Process," AIChE J. , 65, 7, pp. 1-11 (2019)

Current and Future Work

- Advanced Step Moving Horizon State Estimation
- Embedded discrete decisions for nonsmooth dynamics
- Multi-stage Stochastic formulations for NMPC with uncertainties and recourse variables
- Structured Dynamic Decompositions for Newton Steps in IPOPT
- Larger, more challenging applications
  - Big data in MHE (spectral measurements)
  - PDEs as process models
  - Exploit multiple time scales
- CAPRESE: Python/Pyomo framework for asNMPC/asMHE and Sufficient horizon lengths found via sIPOPT