Onboard Trajectory Planning for a New Class of Hybrid Aircraft

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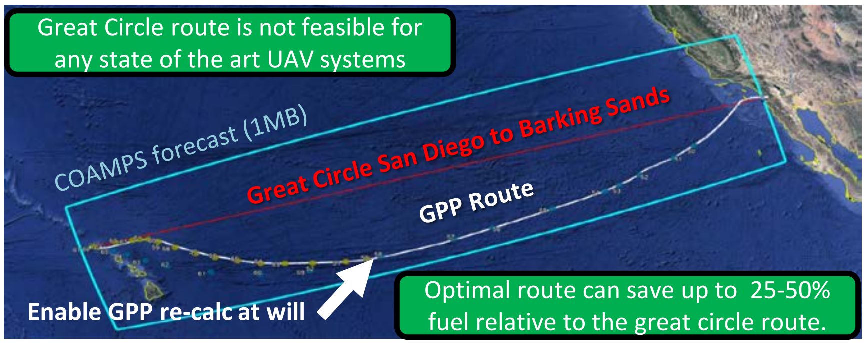
Outline

- Motivation – energy efficiency
- Approach:
  - *intelligent control
  - a/c design & new instrumentation & novel propulsion
- OC task formulation & control synthesis
- Results:
  - “Synthetic” weather
  - COAMPS weather
- Computational bottlenecks:
  - Existing solutions
  - Desired but Missing “pieces”
- What is achievable today

Motivation – "Multi-Day Endurance of a Group 2 UAS Utilizing Pacific Energy Sources"

Motivation – enhance current mission effectiveness via advanced energy behavior (DOD Operational Energy)



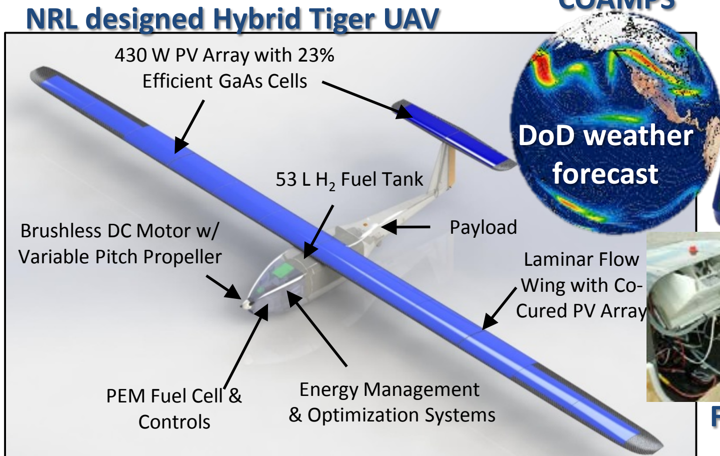
Objective – advance operational energy strategy:

- Increase future UAV capability via adaptable use of various sources of energy
- Enhance current mission effectiveness via predictive energy forecast and optimal routing
- Identify and reduce risk of energy shortage via robust adaptive mission replanning and intelligent control

Concept – demonstrate synergistic range and endurance benefits by integrating fuel cell propulsion, soaring, solar harvesting, and optimal path planning.

Approach – integrate the latest advances in energy storage, harvesting, and recovery technologies in the novel onboard software capable of rapid *energy optimal global path planning (GPP).*

NRL designed Hybrid Tiger UAV

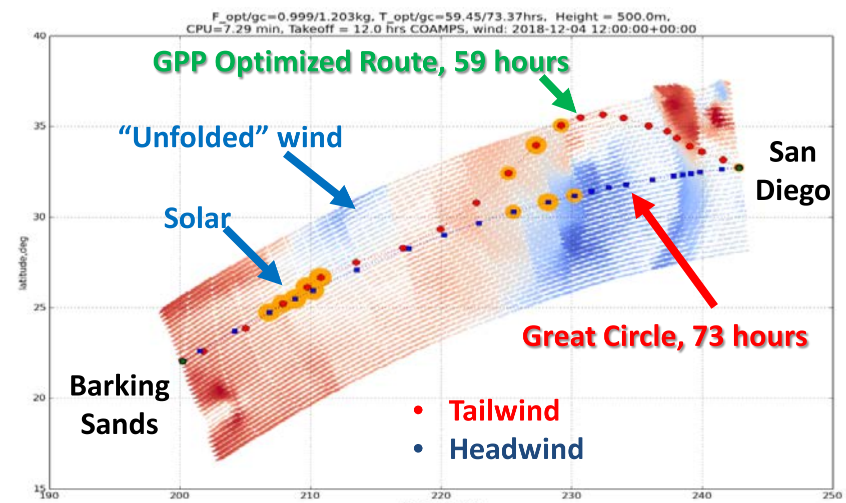


COAMPS

Miniature CPU

Fuel Cell & H2 Fuel Storage

Solution – minimum energy/fuel solution obtained by utilizing classical Pontryagin optimal control approach.
Key Deliverable - previously not feasible routes (CA-HI) can be optimally flown and rapidly recomputed onboard.



Patent: U.S. PTO 16/155,968, U.S. PCT PCT/US18/55144

Existing Constraints & Desired Features

Desired features of the GPP solution:

- Completeness - need qurantees of complete exploration of given domain
- Optimality – need analytical guarantees of optimality of the solution
- Feasibility – practically feasible to implement onboard
  - feasibility of CPU load
  - feasibility of memory allocation
  - ability to monitor the solver as it runs

Constraints of many of the existing methods:

- NO closed-form solution for complex dynamics of aircraft
- FAIL in complex and dynamic environment (time-varying wind, obstacles, etc)
- INFEASIBLE for online onboard implementation
- LACK of convergence for stiff ODEs
- Initial guess - problematic

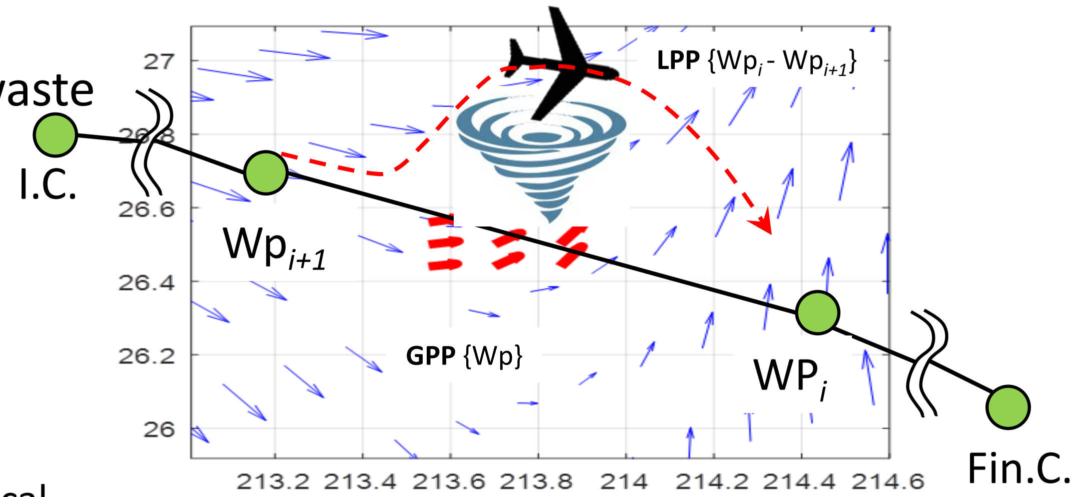
- "SURVEI OF NUMERICAL METHODS FOR TRAJECTORY OPTIMIZATION," by John Betts, Journal of Guidance, Control, and Dynamics, vol. 21, #2, March-April 1998.
- "OPTIMAL CONTROL AND NUMERICAL SOFTWARE: AN OVERVIEW" by H.S. Rodrigues, M.T. Monteiro, D.M. Torres, in 'Systems Theory: Perspectives, Applications and Developments', Nova Science Publishers, Editor: Francisco Miranda, Jan 2014.
- "A SURVEY OF NUMERICAL METHODS FOR OPTIMAL CONTROL" by Anil V. Rao, AAS 09-334
- "OPTIMAL PATH PLANNING AND POWER ALLOCATION FOR LONG ENDURANCE SOLAR-POWERED UAV," by S.Hosseini, R.Dai, M. Mesbahi, in proceedings of ACC2013, Washington, DC, June 17-19, 2013

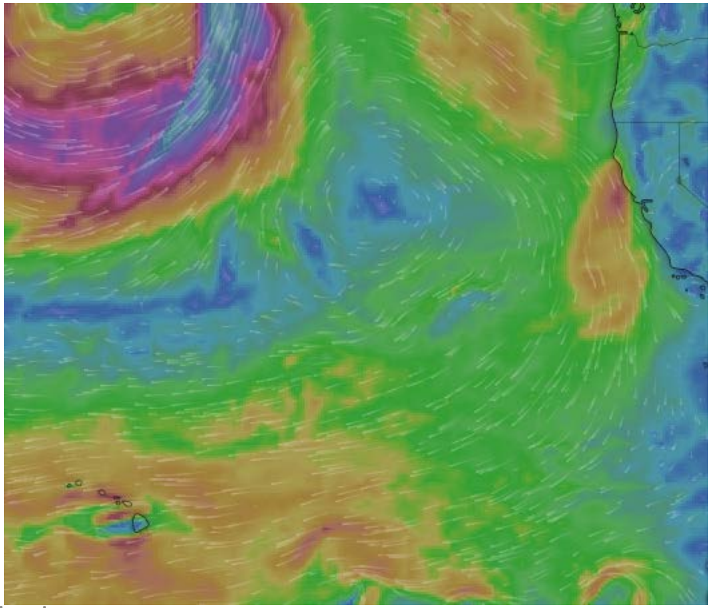
Mission Planning

- Practical objective – maximum endurance by minimizing the waste of constrained energy resource onboard

- Challenges:
  - Weather:
    - time-varying weather = {wind, local thermals, solar irradiance}
    - stochastic nature diverging with time
    - limited fidelity
  - Limited communication and computational resources

- Approach:
  - Global route – GPP
  - Local route – LPP
  - GPP is a “reference to follow” for the LPP



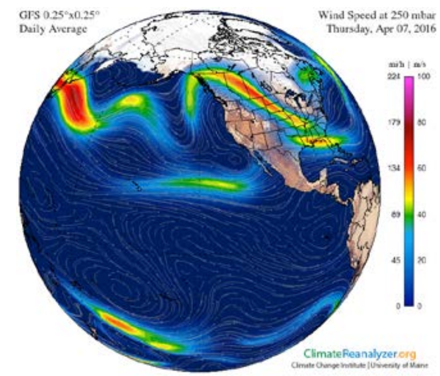


https://www.windy.com/?900h,33.761,-132.671,4

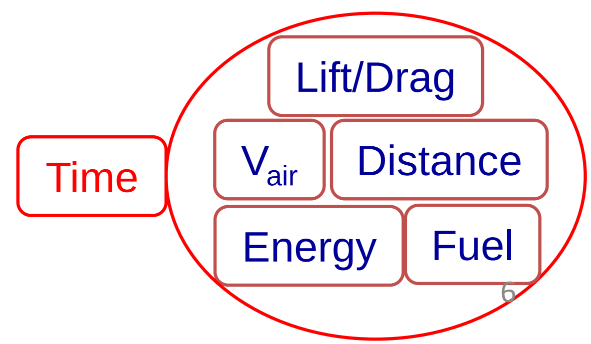
Key Components of GPP

- **Aircraft**
  - Aerodynamics C_L , C_D
  - Fuel consumption dynamics vs thrust
  - Solar efficiency
  - “Battery” efficiency (minor now, big potential)

- **Weather prediction model** – COAMPS NRL/MRY
  - 3D wind components as functions of LLA & Time
  - Solar flux => essential chunk of energy
  - PBL => essential chunk of hybrid power
  - Variability of weather => confidence



- **Time**
  - Defines non-autonomous nature
  - Optimizes the entire mission => start of the mission
  - “Convolution parameter” of Energy&Dynamics of flight



Plant Model

- Aircraft :

$$\dot{x} = V\cos\psi + W_x(x,y,t)$$
$$\dot{y} = V\sin\psi + W_y(x,y,t)$$
$$\dot{\psi} = \frac{g\tan\varphi}{V_g}; V_g = \sqrt{\dot{x}^2+\dot{y}^2}$$

$$u_1 = \varphi(t) - \text{ bank angle}$$
$$u_2 = V(t) - \text{ airspeed}$$

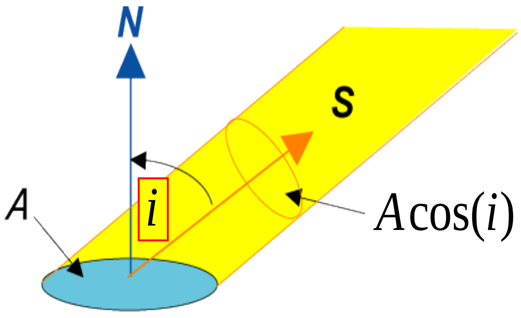
- Power loss due to drag

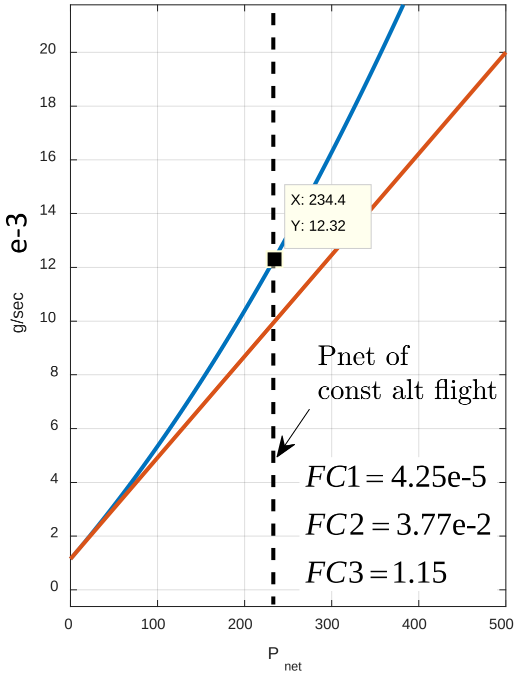
$$\left.\begin{aligned} C_L &= \frac{2W}{\rho V^2 S\cos\varphi} \\ C_D &= C_{D_0} + K\cdot C_L^2 \\ D &= \frac{\rho V^2}{2} S\cdot C_D \\ T &= D \\ P_{drag} &= \frac{T\cdot V}{\eta_{prop}} \end{aligned}\right\} \quad E_{drag} = \int_0^{t_f} P_{drag}(\varphi, V, t)\,dt$$

- Separating controls (φ and V) from the parameterized model

$$P_{drag} = \frac{T\cdot V}{\eta_{prop}} = \frac{\rho V^3}{2\eta_{prop}} S\cdot\left(C_{D_0} + K_p\cdot C_L^2\right) =$$
$$= \frac{\rho V^3}{2\eta_{prop}} S\cdot C_{D_0} + \frac{\rho V^3}{2\eta_{prop}} S\cdot K_p\cdot C_L^2 = \frac{\rho V^3}{2\eta_{prop}} S\cdot C_{D_0} + \frac{\rho V^3}{2\eta_{prop}} S\cdot K_p\cdot\frac{4W^2}{\rho^2 V^4 S^2\cos^2\varphi} =$$
$$= \frac{\rho\cdot S\cdot C_{D_0}}{2\eta_{prop}}\cdot V^3 + 2\frac{K_p S\left(W/S\right)^2}{\rho\eta_{prop}}\cdot\frac{1}{V\cos^2\varphi} = K_{p1}\cdot V^3 + K_{p2}\cdot\frac{1}{V\cos^2\varphi}$$

Plant Model





- Power gain due to solar photovoltaics

$$\cos(i) = \cos(\varphi)\sin(\mathrm{El}) - \cos(E)\sin(Az - \psi)\sin(\varphi)$$

$Az(t), El(t) -$ azimuth and elevation of Sun

$$P_{solar} = \eta_{solar} P_{sflux} A_{PV} \cos(i)$$

$$E_{solar} = \int_{t_{start}}^{t_f} P_{solar}(\varphi, V, t)\ dt$$

- Fuel consumption model

$$P_{net}(\varphi, V, t) = \left| P_{drag} - P_{solar} \right|$$

$$\dot{m}_F = FC1 \cdot P_{net}^2 + FC2 \cdot P_{net} + FC3$$

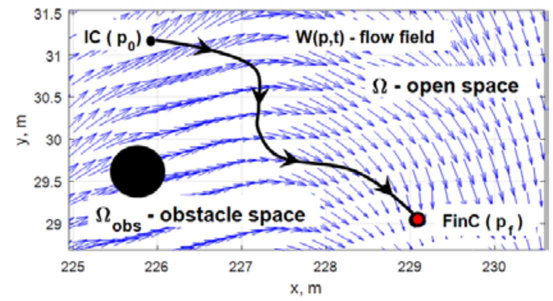
$$m_F = \int_{t_{start}}^{t_f} \dot{m}_F(\varphi, V, t)\ dt$$

δ=0.2e-3 => ε =172 g over 24 hours period=> 17 hours of flight; ~10g/hour

Minimum Energy Optimum Control

Find the optimal airspeed V^* and bank angle φ^* control functions that minimize

$$J^* = \min_{\varphi^*, V^*} \int_{t_0}^{t_f} P_{net} dt$$



subject to the states and costates dynamics

$$\dot{x} = V\cos(\psi) + W_x(x,y,t)$$
$$\dot{y} = V\sin(\psi) + W_y(x,y,t)$$
$$\dot{\psi} = g\tan(\varphi)/V_g, V_g = \sqrt{\dot{x}^2 + \dot{y}^2}$$
$$\dot{\tau} = 1, \quad \tau = t/t_f$$

$$\dot{\lambda}_x = -\partial H/\partial x = -\lambda_x \frac{\partial W_x(x,y,t)}{\partial x} - \lambda_y \frac{\partial W_y(x,y,t)}{\partial x} - \lambda_\psi \frac{\partial \dot{\psi}}{\partial x}$$
$$\dot{\lambda}_y = -\frac{\partial H}{\partial y} = -\lambda_x \frac{\partial W_x(x,y,t)}{\partial y} - \lambda_y \frac{\partial W_y(x,y,t)}{\partial y} - \lambda_\psi \frac{\partial \dot{\psi}}{\partial y}$$
$$\dot{\lambda}_\psi = -\frac{\partial H}{\partial \psi} = \lambda_x V\sin(\psi) - \lambda_y V\cos(\psi) + K_s\sin(\phi)\cos(e)\cos(a-\psi) - \lambda_\psi \frac{\partial \dot{\psi}}{\partial \psi}$$
$$\dot{\lambda}_\tau = -\frac{\partial H}{\partial \tau} = -\lambda_x \frac{\partial W_x(x,y,t)}{\partial \tau} - \lambda_y \frac{\partial W_y(x,y,t)}{\partial \tau} - \lambda_\psi \frac{\partial \dot{\psi}}{\partial \tau}$$

, where

$$H = V^3 K_{p1} + K_{p2}/V cos^2(\varphi) - K_s\cos(\theta_i) + \lambda_x\dot{x} + \lambda_y\dot{y} + \lambda_\psi\dot{\psi} + \lambda_\tau$$

and the associated boundary conditions

$$x(t_0) = x_0;\ y(t_0) = y_0$$
$$x(t_f) = x_f;\ y(t_f) = y_f$$
$$\psi(t_0) = \psi_0;\ \psi(t_f) = \psi_f$$
$$\tau(t_0) = 0; \tau(t_f) = 1$$

$$+ \quad H(x, y, \psi, \tau, \lambda_x, \lambda_y, \lambda_\psi, \lambda_\tau)_{t_f} = 0$$

Synthesis

A1: Aaircraft is equipped with a **stabilizing autopilot** that effectively eliminates the nonlinear flight dynamics from consideration.
A2: The bank angle φ is small.

$$\frac{\partial H}{\partial \varphi} = \frac{2K_{P2}}{V}\tan\varphi(\mathrm{tg}^2\varphi+1) + K_s\sin\varphi\left(\sin(e)+\frac{\cos(e)\sin(a-\psi)}{\tan\varphi}\right)+\lambda_\psi\frac{g}{V_g}\frac{1}{\cos^2\varphi}=0$$

$$\frac{\partial H}{\partial V} = 3V^2K_{P1}-\frac{1}{V^2}K_{P2}(\tan^2\varphi+1)+\lambda_x\cos\psi+\lambda_y\sin\psi=0$$

Results:

$$\tan(\varphi^*) = -\frac{V}{V_g}\frac{\lambda_\psi g + V_g K_s\cos(e)\sin(a-\psi)}{2K_{P2}+V\cdot K_s\sin(e)}$$

$$V^{*2} = \sqrt{\boxed{\frac{4K}{3\rho^2C_{D0}}\left(\frac{mg}{S}\right)^2}+\frac{\eta_{prop}}{18\rho SC_{D0}}\Lambda^2}-\frac{\eta_{prop}}{3\rho SC_{D0}}\Lambda$$

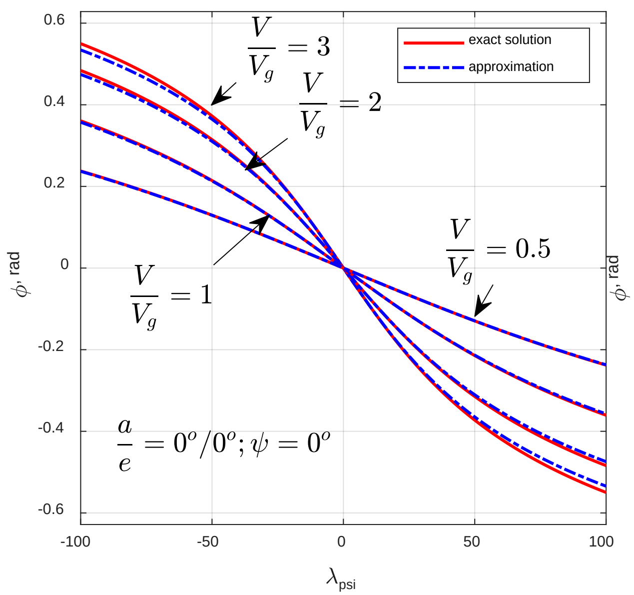
$$\Lambda = \lambda_x cos\psi + \lambda_y sin\psi$$

$$V_{minP}^2 = \frac{2mg}{\rho S}\sqrt{\frac{K}{3C_{D0}}}$$

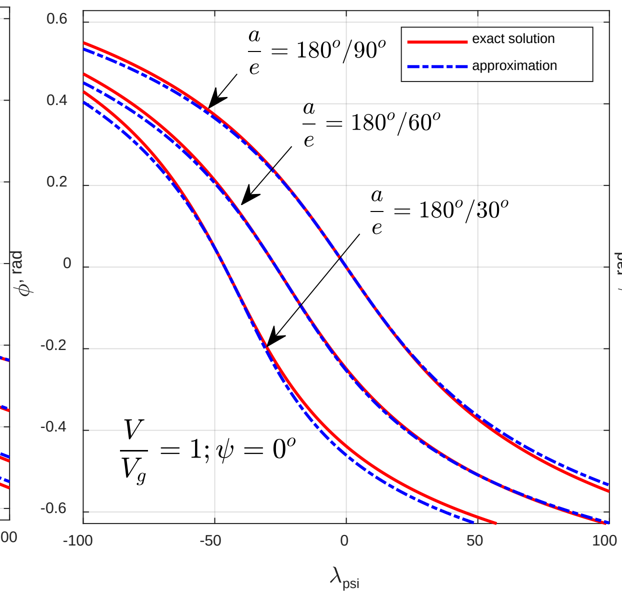
the optimal speed to fly for the minimum required power V_{minP} in horizontal flight

OC Analysis

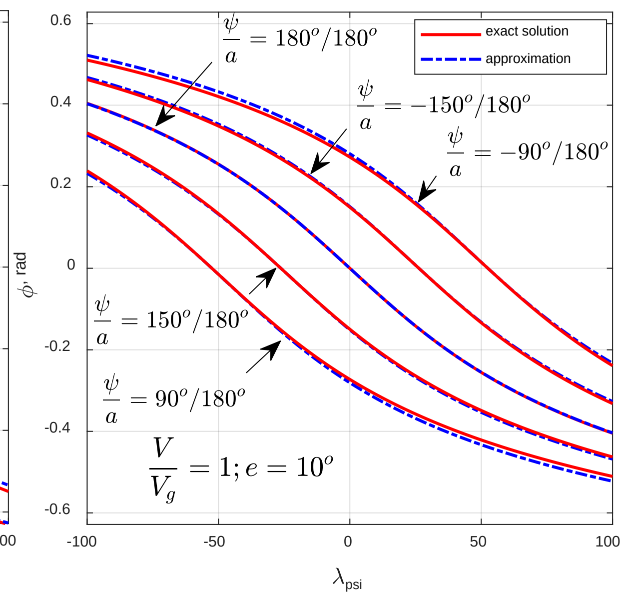
$$\tan(\varphi^{*}) = -\frac{V}{V_g}\frac{\lambda_{\psi} g + V_g K_s \cos(e)\sin(a - \psi)}{2K_{P2} + V \cdot K_s \sin(e)}$$



a. airspeed



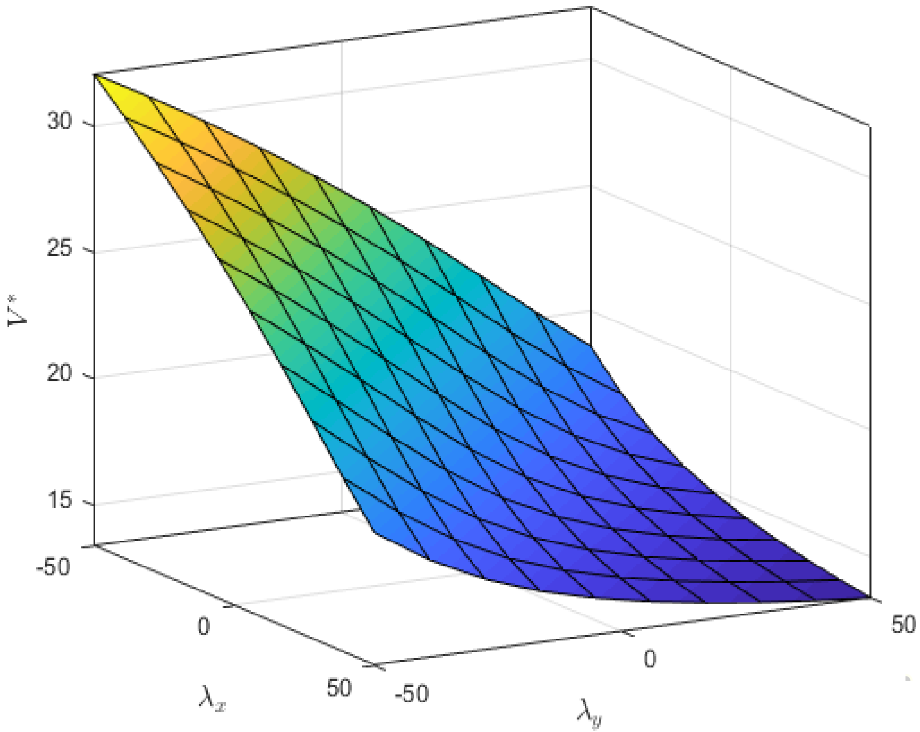
a. sun elevation

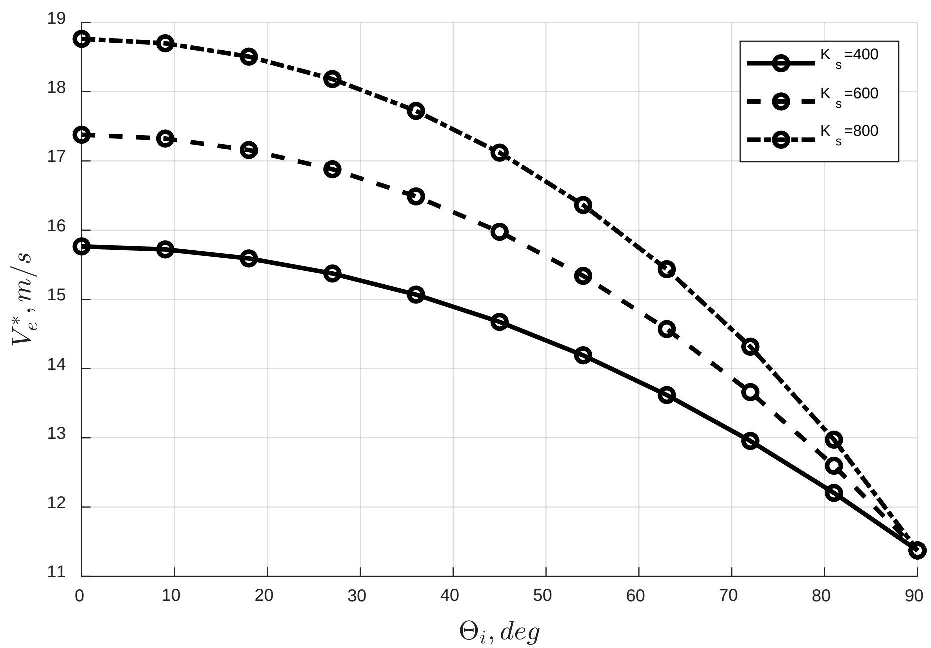


a. sun azimuth

OC Analysis

$$V^{*2} = \sqrt{\frac{4K}{3\rho^2 C_{D0}}\left(\frac{mg}{S}\right)^2 + \frac{\eta_{prop}}{18\rho S C_{D0}}\Lambda^2} - \frac{\eta_{prop}}{3\rho S C_{D0}}\Lambda$$



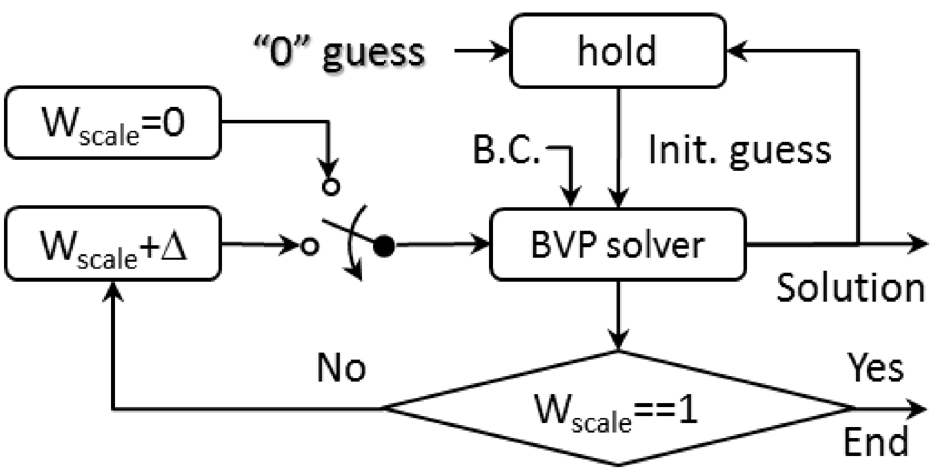


Solving BVP

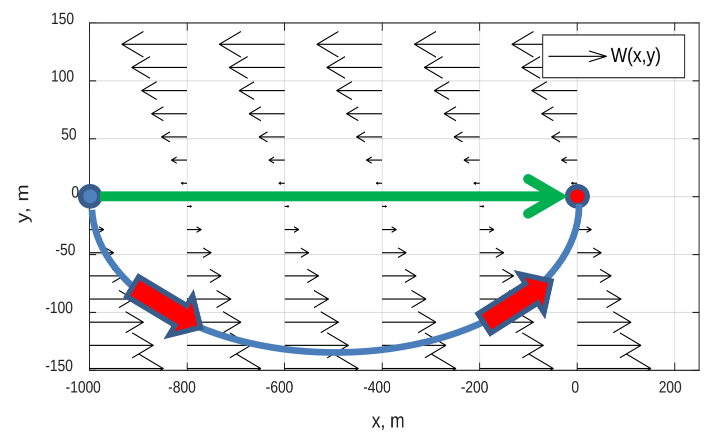
- Challenge - sensitivity to the initial guess:
  - Scaling
  - Continuation
  - Homotopy methods

  } design and solve a sequence of problems starting with a trivial one, and then use the previous solution as an initial guess for the next one.

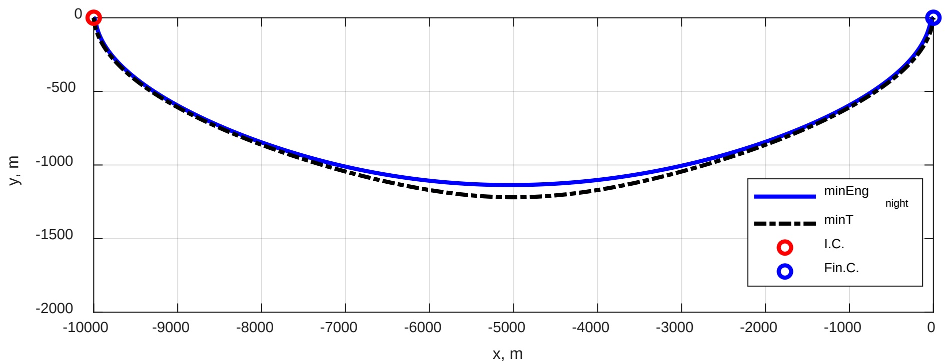
- Solution – when there is no wind => $W_{\text{scale}} = 0$ =>the resulting trajectory is necessarily a straight line



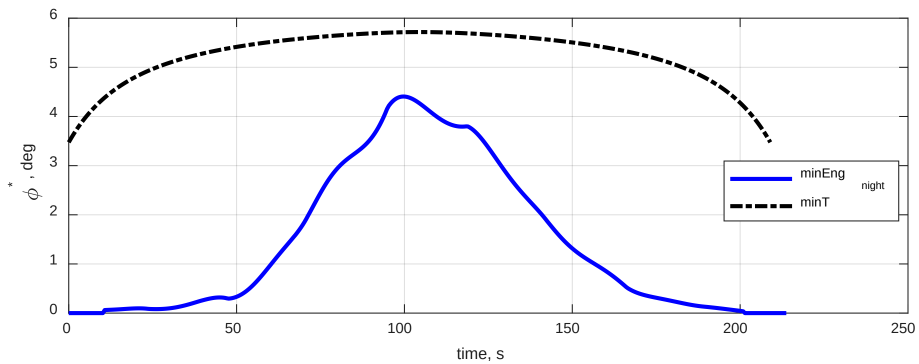
$W_x(x, y, t) = -h_y \cdot y, W_y(x, y, t) = 0$



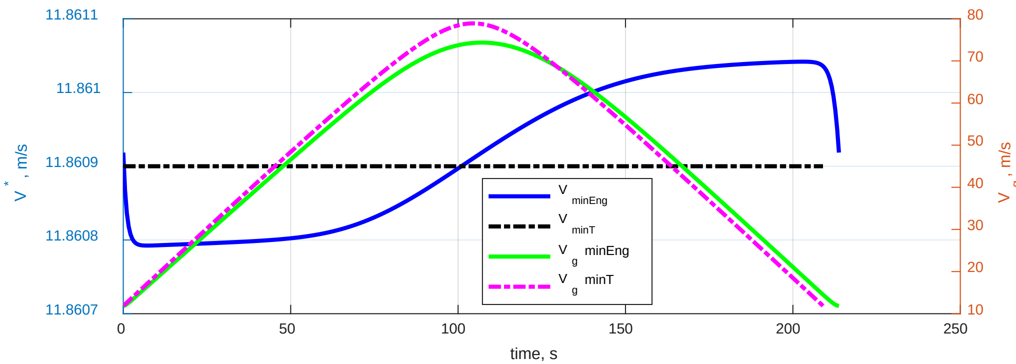
Synthetic Wind :: Night



a – trajectories



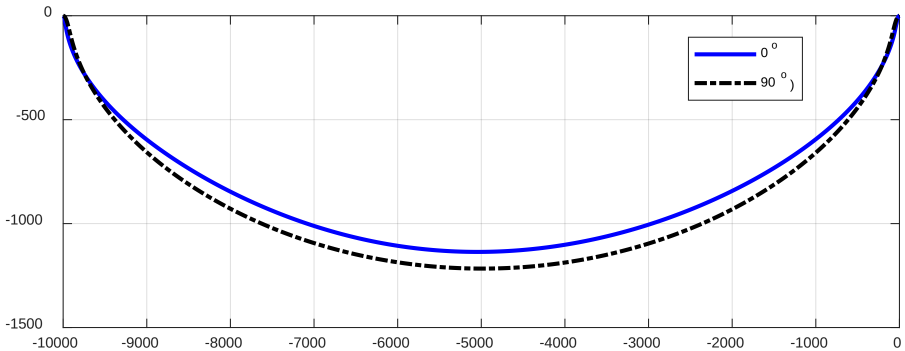
b – ϕ control



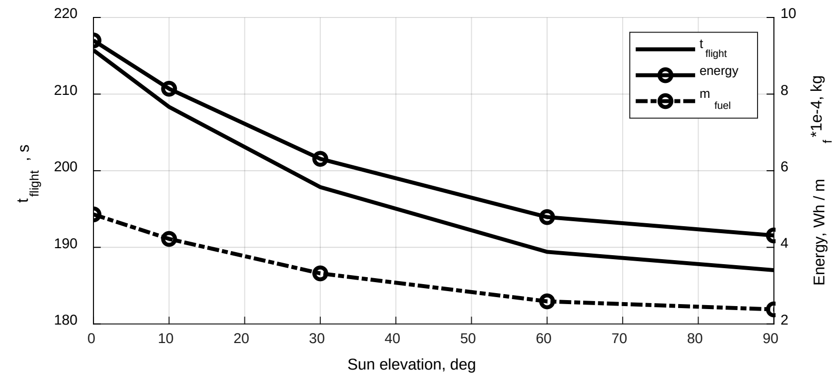
c – the airspeed *V* and the ground speed *Vg*

- Synthesized OC closely approximates *minT* solution of Zermelo task
- BVP solver is fast => seconds

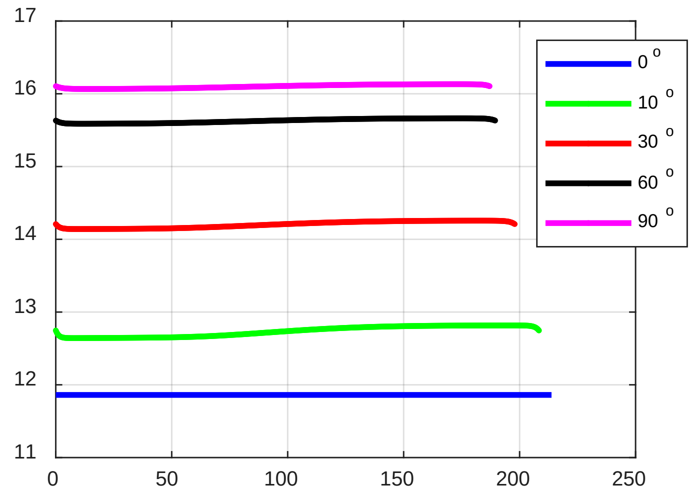
Synthetic Wind :: Day



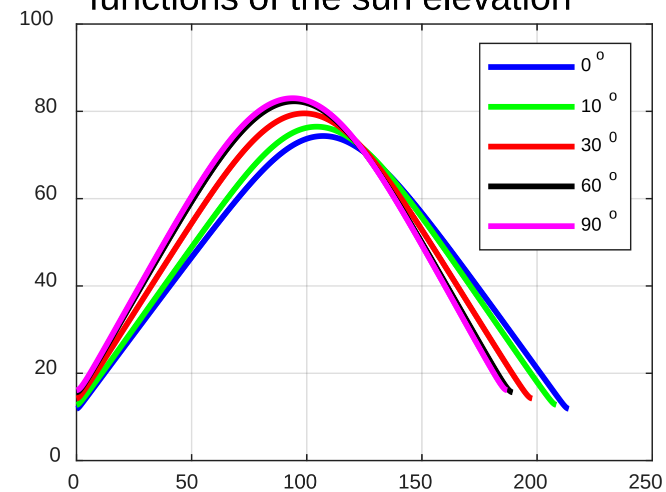
a – trajectories



d - Energy and flight time as functions of the sun elevation



b – *the airspeed*

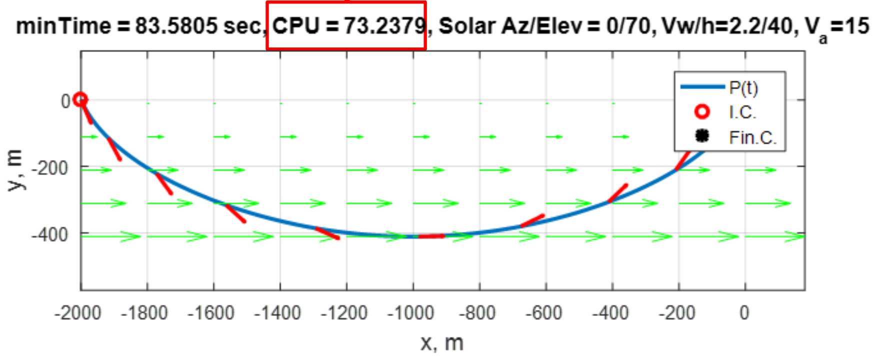


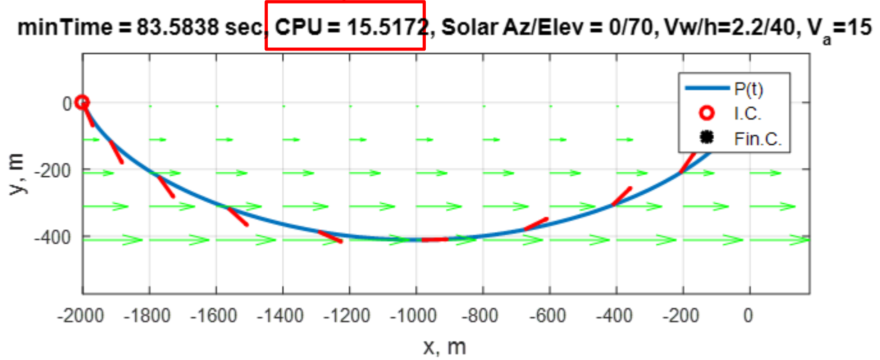
c – the ground airspeed *Vg*

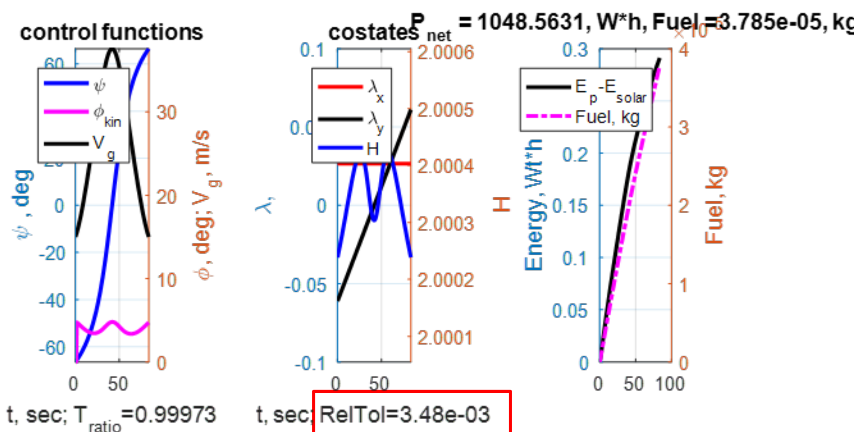
- Synthesized OC is sufficiently different however can be ‘initialized’ by *minT*

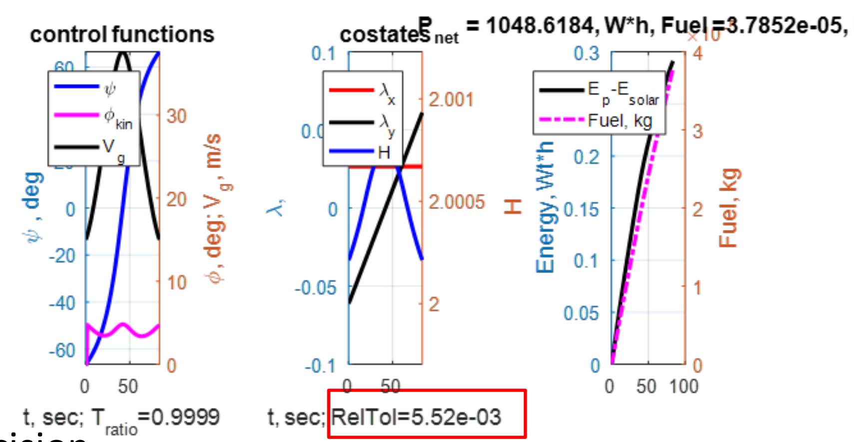
Single Shot vs Continuation (BVP5C)

CPU time reduced by ~4.7 times









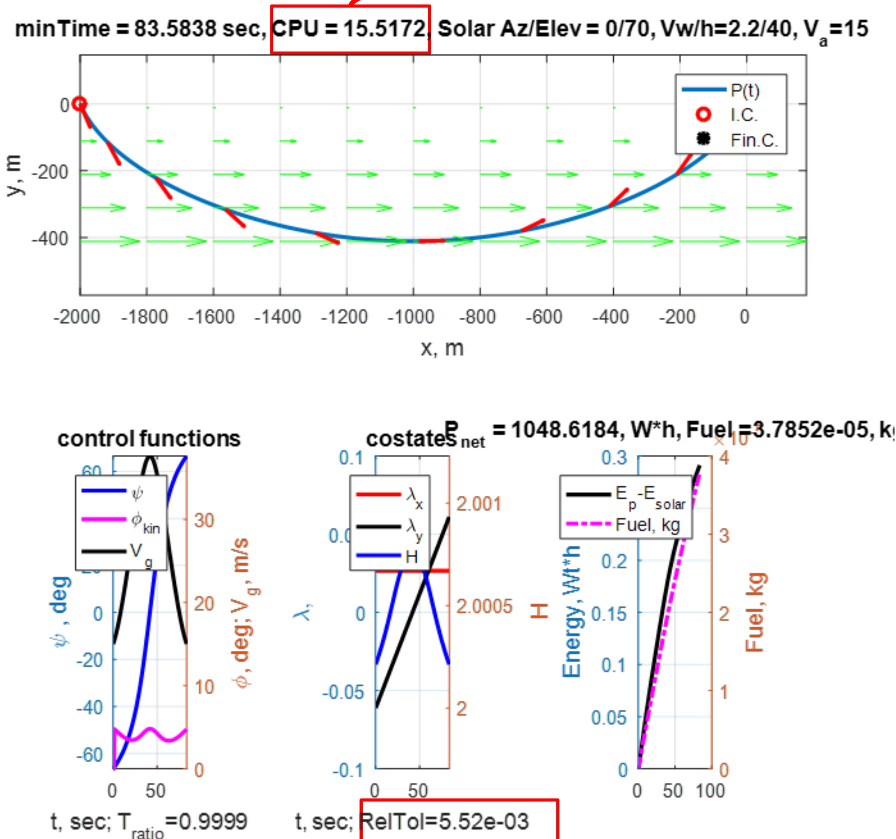
Same precision

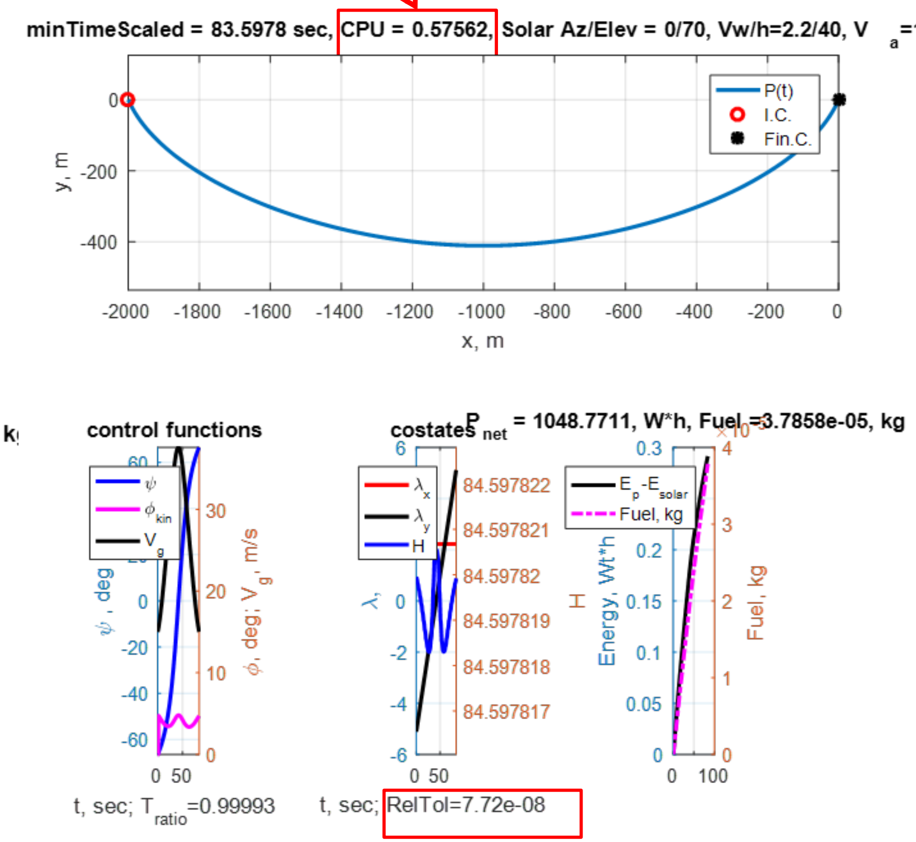
Single shot

Continuation

Continuation vs Scaling+Continuation (BVP5C)

CPU time reduced by 26 times





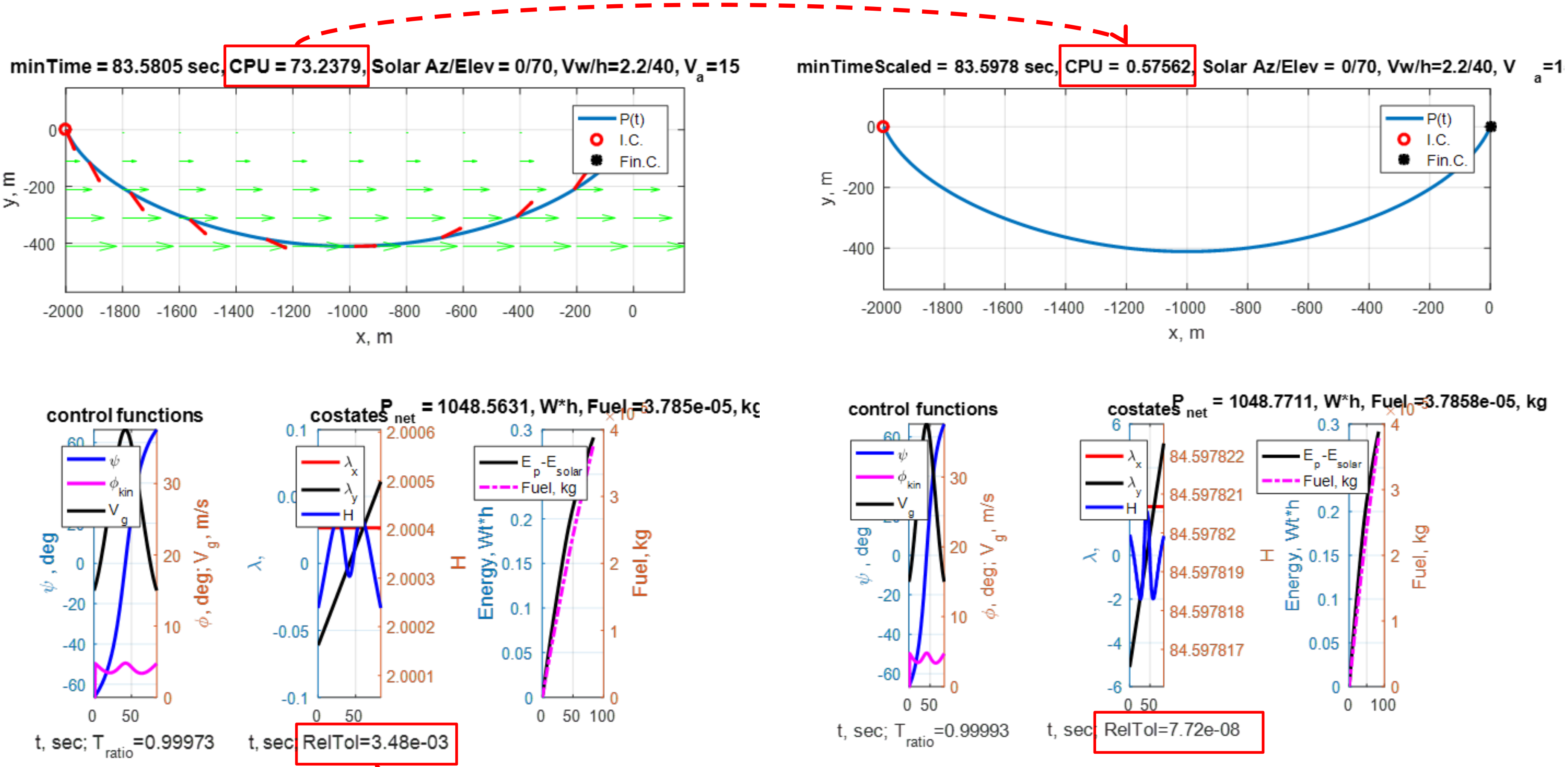
Improve precision

Continuation

Continuation +Scaling

Single Shot vs Scaling+Continuation (BVP5C)

CPU time reduced by 127 times



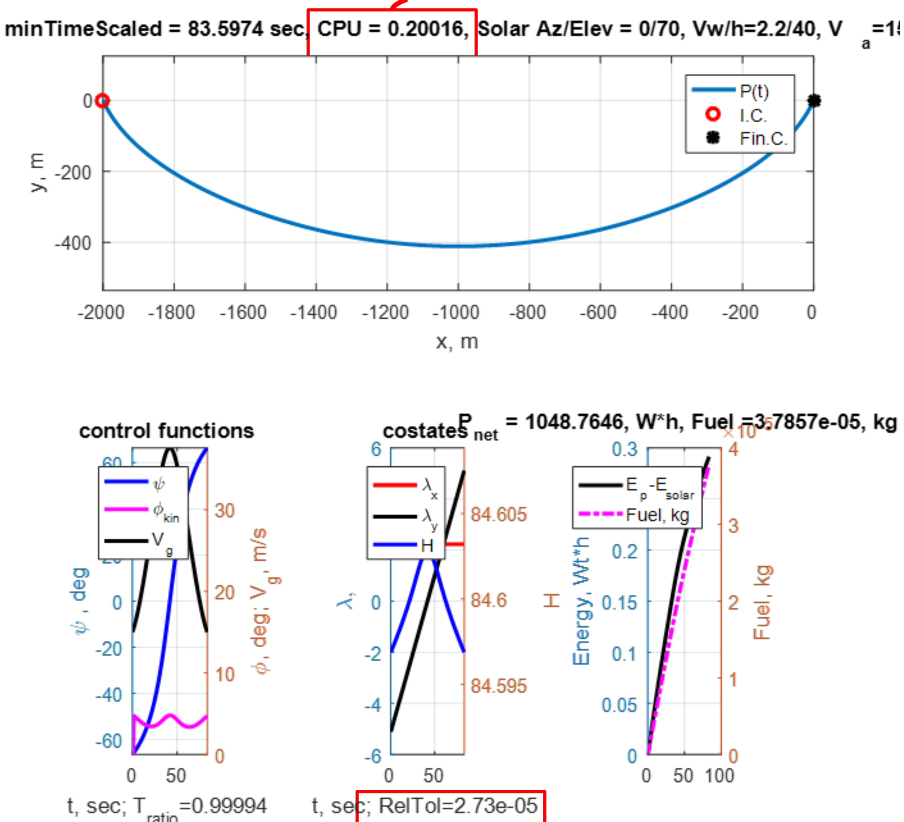
Improve precision

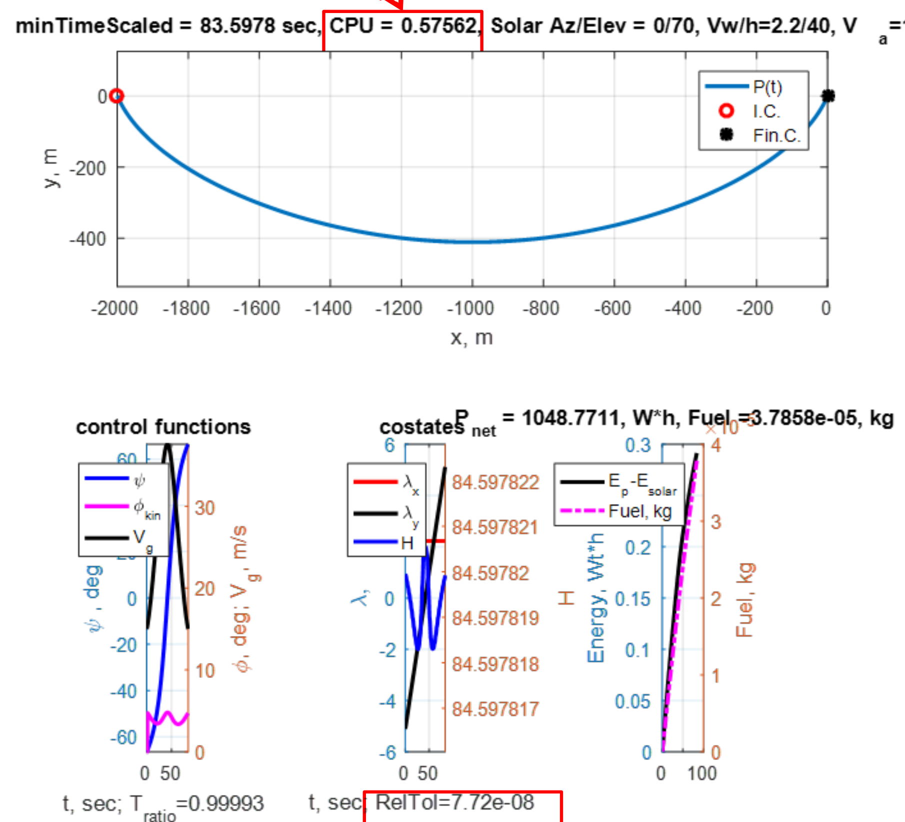
Single shot

Continuation +Scaling

BVP4C vs BVP5C in Scaling+Continuation

CPU time is ~2 times less





Precision is the price,
however "1e-3" is more than enough

Continuation +Scaling

Continuation +Scaling

Discretization = 100, the same for both methods

Analysis

NOTE – keep in mind that there is **NO interpolation**

- Continuation:
  - improves convergence properties of bvp solver by parameterizing the task when a single shot solution is not guaranteed.
  - significantly simplifies the choice of an initial guess of states and co-states of the dynamic system.
  - reduces the computational time by a factor of ~ 5.
- Scaling of ODE
  - Reduces sensitivity of the BVP solver to nonlinearities.
  - Reduces the computational time by a factor of ~20.
- Combination of ODE Scaling and Continuation:
  - Combines the benefits of both.
  - Reduces computational time by more than a factor of ~100.

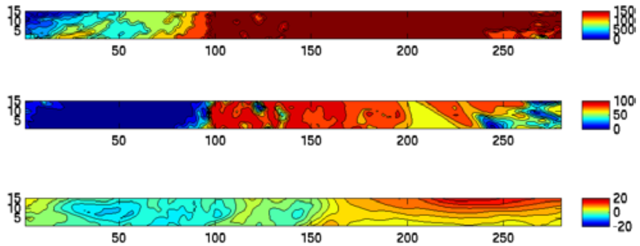
- What is next:
  - Use Scaling+Continuation result of minTime task as an initial guess of minEnergy & minFuel. Already prototyped with good promise.
  - Solve the GPP task with interpolated COAMPS data.

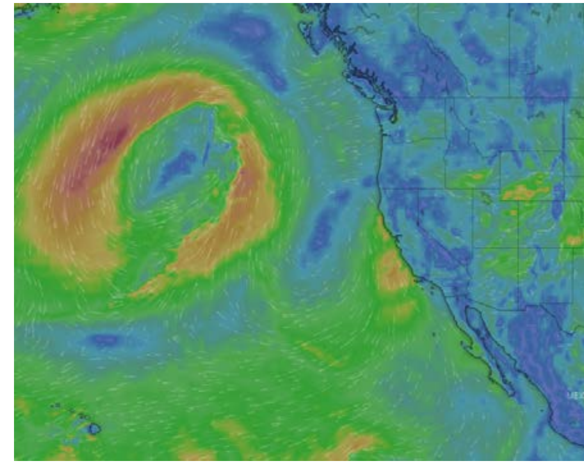
COAMPS Wind

PBL Height

Relative Humidity 1.5 km ASL

Zonal Wind 3 km ASL



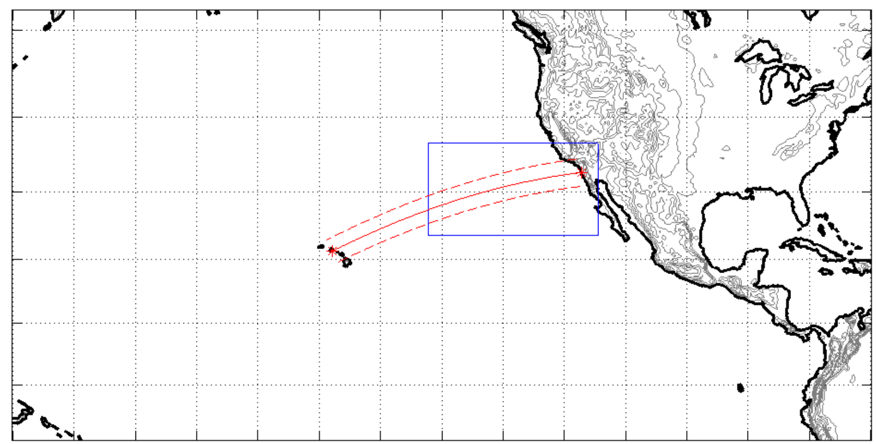


- Height levels: zonal, meridional, vertical winds, relative humidity
- Single levels: PBL height, incoming solar radiation, theta star (buoyancy meassure)

70Mb/72hour

Solid red line -- great circle from SND-HNL

- Dashed lines represent a path ~200 km on each side of great circle.
- Path width is adjustable
- COAMPS data interpolated to regular grid along the path



Fast interpolation of vectorized queries is one of the heaviest computational tasks.

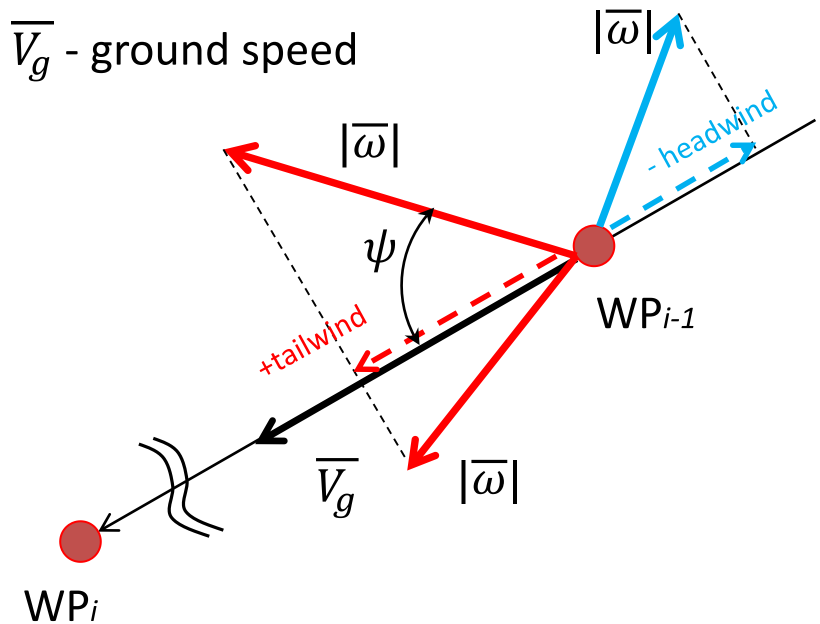
Interpretation od COAMPS-based results

Color-coding should capture:

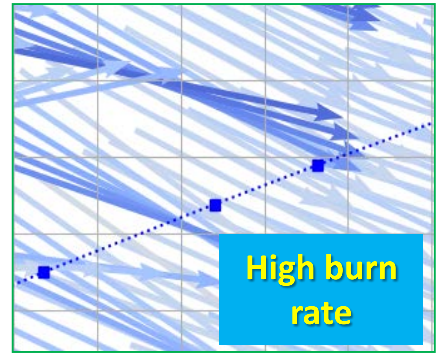
- Magnitude of wind
- Direction of wind w.r.t. the “reference” direction

$|\overline{\omega}|$ - wind

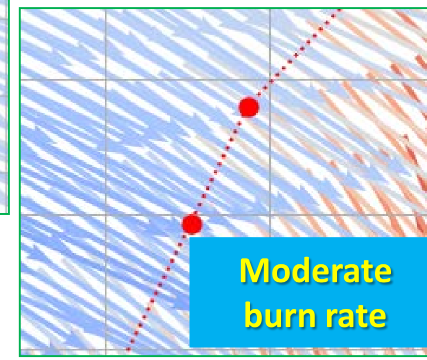
$\overline{V_g}$ - ground speed



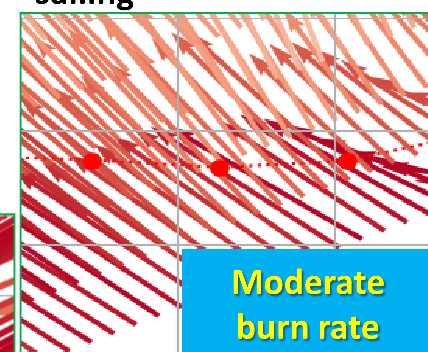
Strong headwind



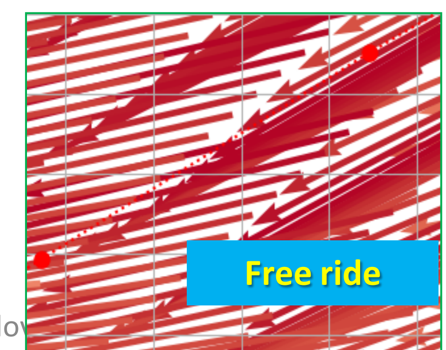
Moderate crosswind - sailing



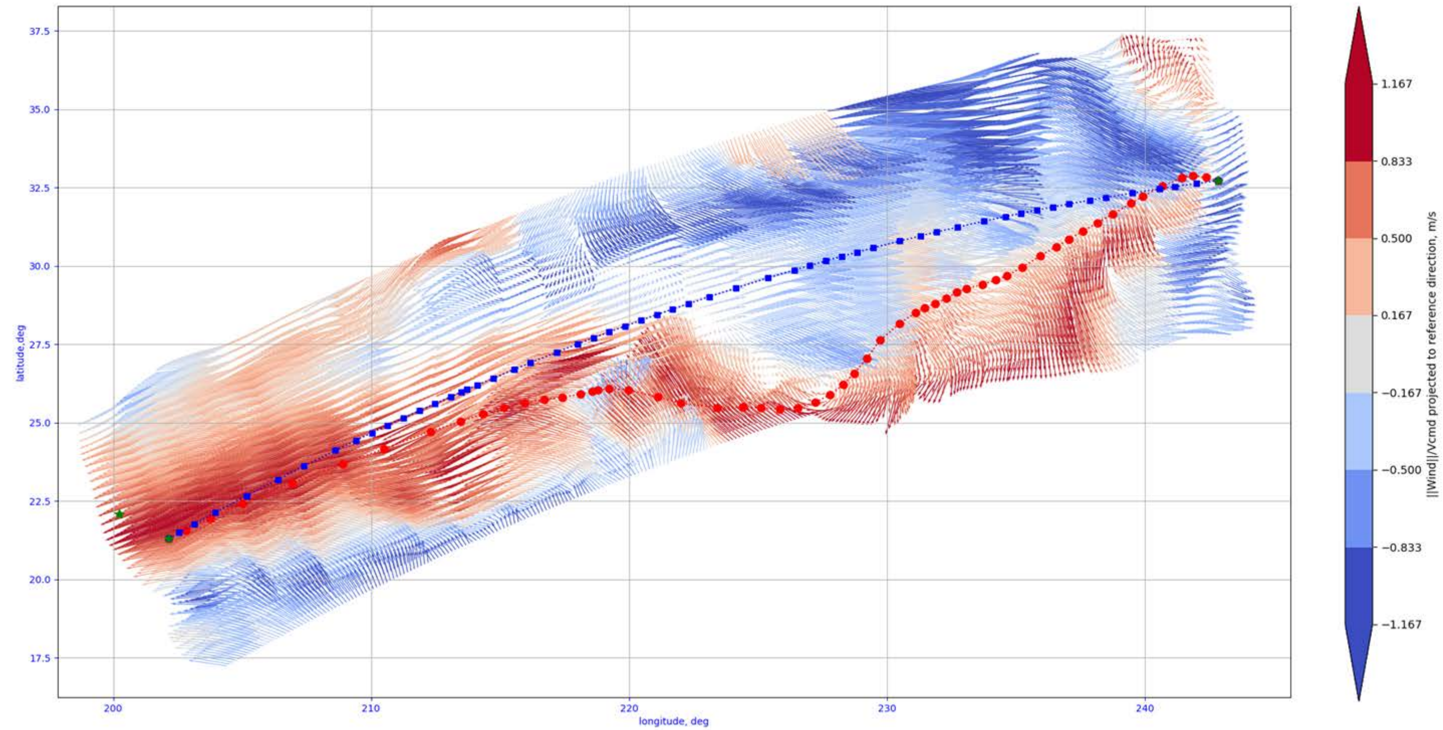
Moderate tailwind - sailing



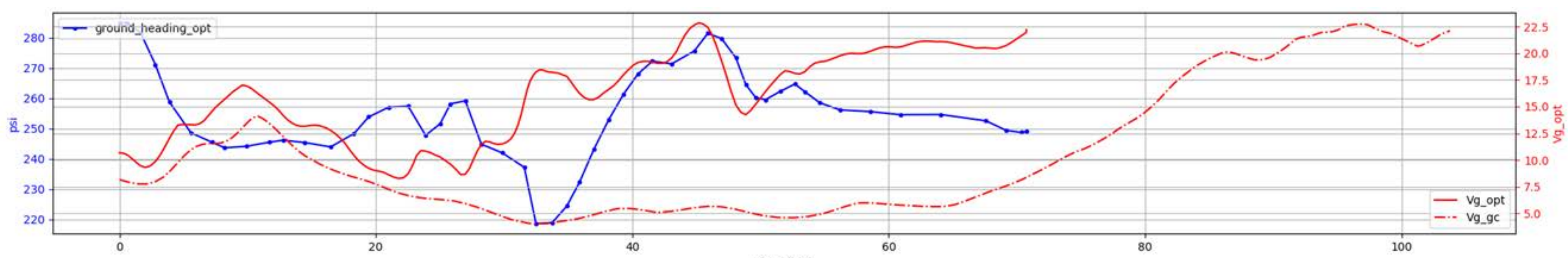
Strong tailwind



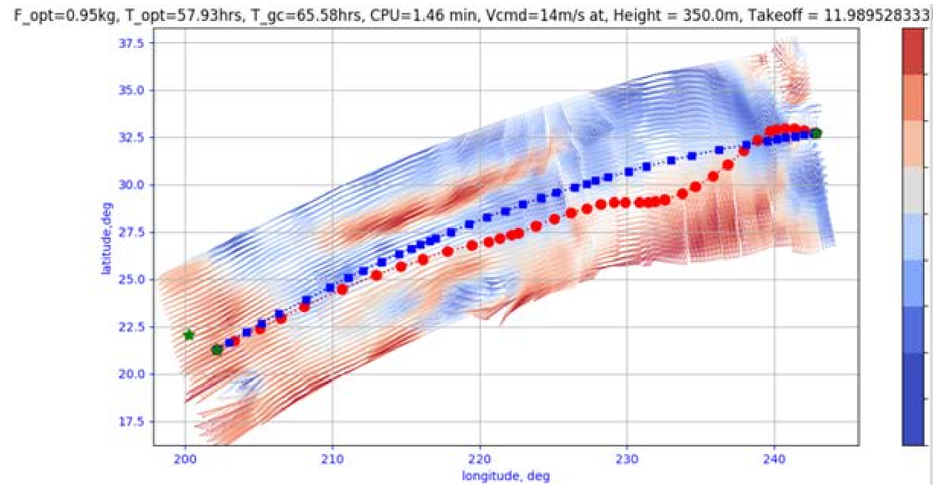
OC over Time-Varying COAMPS

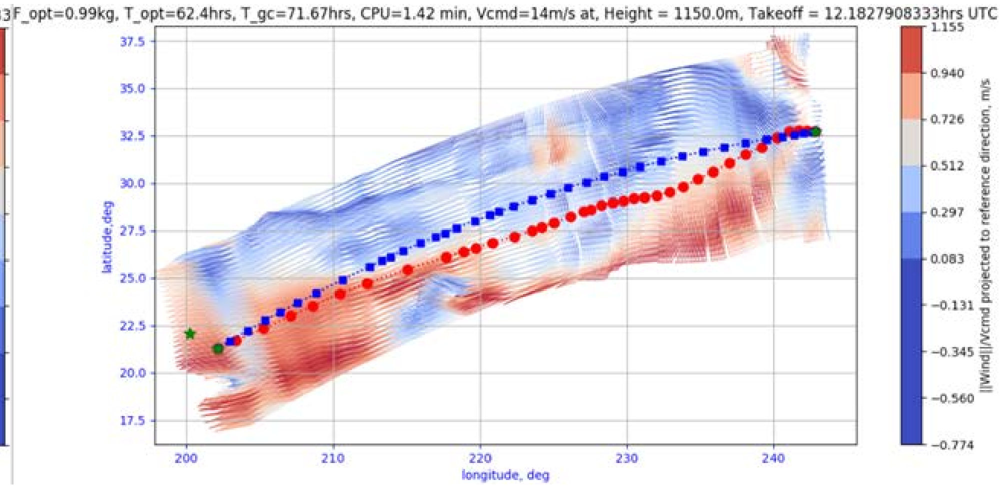


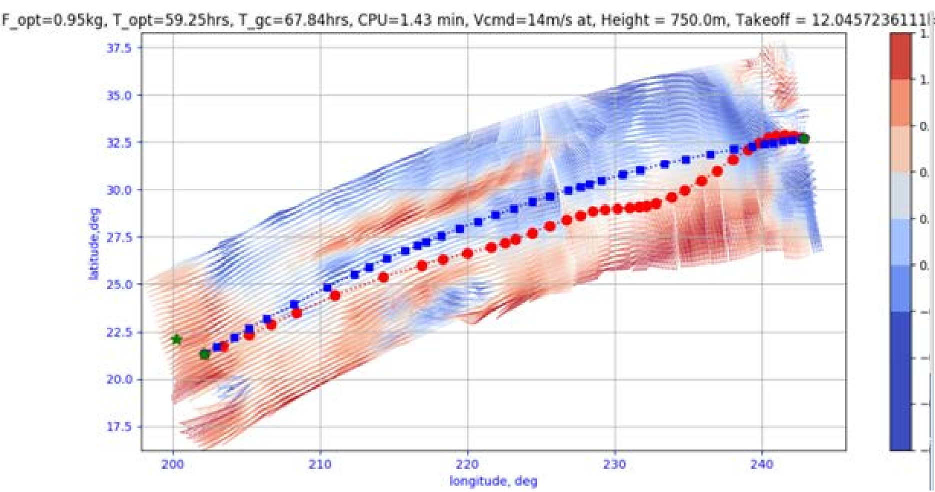
Comparison of the optimal and great circle routes/controls at H=2550 m

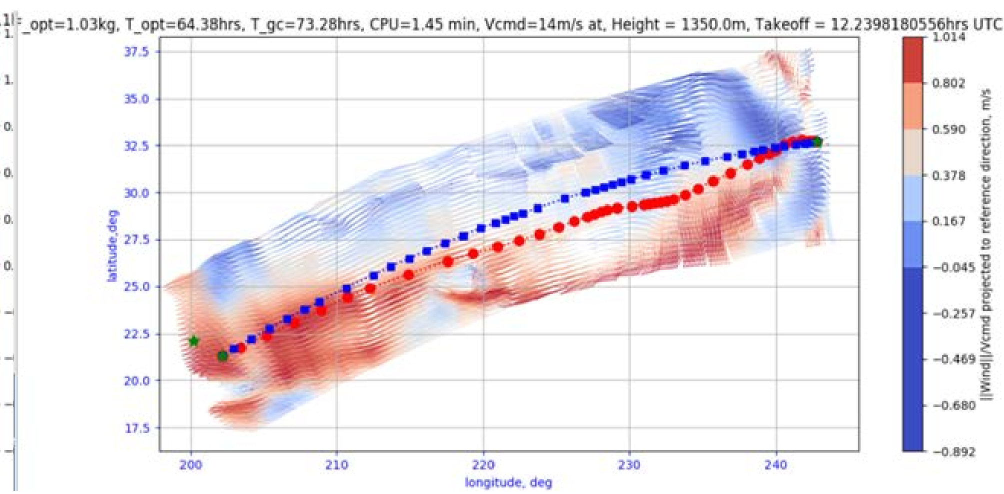


Robustness

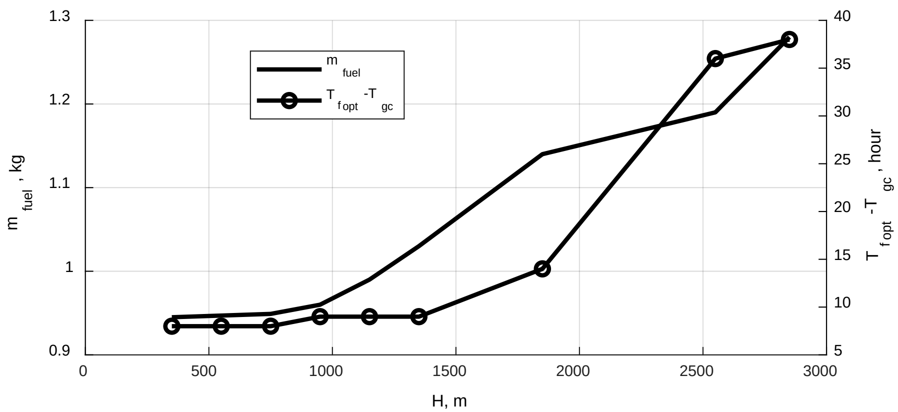

F_opt=0.95kg, T_opt=57.93hrs, T_gc=65.58hrs, CPU=1.46 min, Vcmd=14m/s at, Height = 350.0m, Takeoff = 11.989528333
latitude, deg
longitude, deg


F_opt=0.99kg, T_opt=62.4hrs, T_gc=71.67hrs, CPU=1.42 min, Vcmd=14m/s at, Height = 1150.0m, Takeoff = 12.1827908333hrs UTC
latitude, deg
longitude, deg
||Wind||/Vcmd projected to reference direction, m/s
1.155
0.940
0.726
0.512
0.297
0.083
−0.131
−0.345
−0.560
−0.774

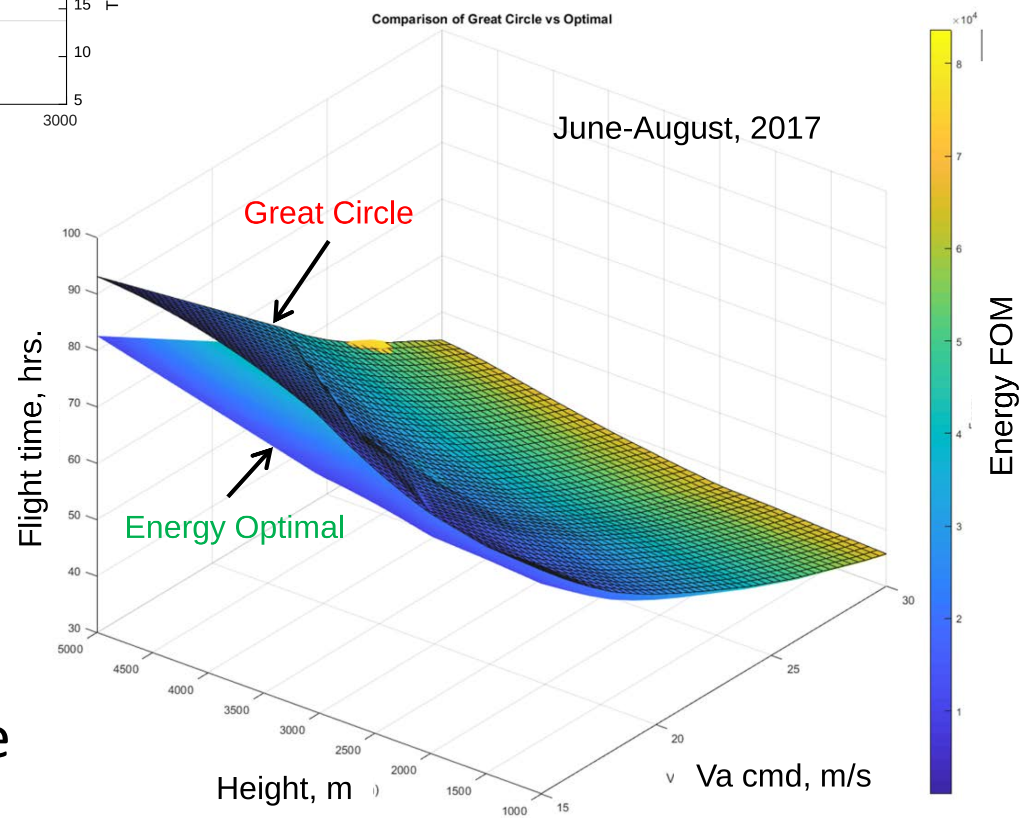

F_opt=0.95kg, T_opt=59.25hrs, T_gc=67.84hrs, CPU=1.43 min, Vcmd=14m/s at, Height = 750.0m, Takeoff = 12.0457236111
latitude, deg
longitude, deg


opt=1.03kg, T_opt=64.38hrs, T_gc=73.28hrs, CPU=1.45 min, Vcmd=14m/s at, Height = 1350.0m, Takeoff = 12.2398180556hrs UTC
latitude, deg
longitude, deg
||Wind||/Vcmd projected to reference direction, m/s
1.014
0.802
0.590
0.378
0.167
−0.045
−0.257
−0.469
−0.680
−0.892

Cumulative Results



- Va_cmd along GC matches the best airspeed for power
- Max achievable Time & Fuel benefits - up to ~**11 hours** or ~**280g** H_2



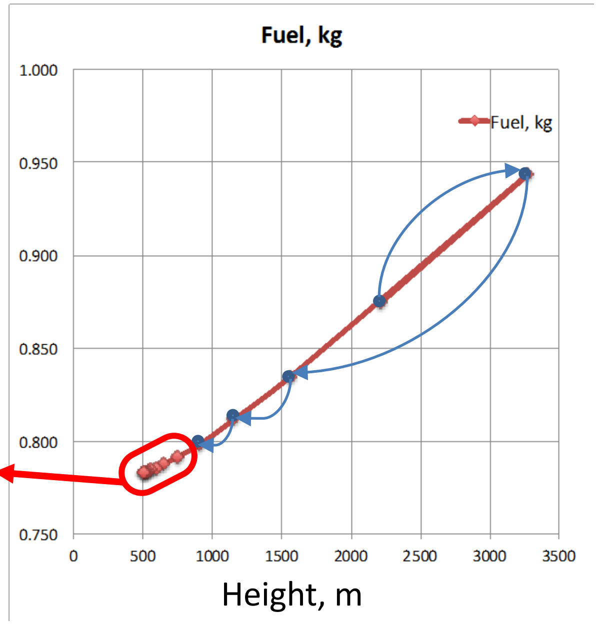
- It is really hard to compete with optimal control !!!

Questions ?

Hurdles of Parallel Processing

Log of Height Search with Fuel Tolerance = 10 gram (Ubuntu i7)

#	Height, m	Cost	Fuel, kg
1	2215.03	3148.08	0.874
2	3274.97	3396.22	0.943
3	1559.95	3003.83	0.834
4	1155.08	2921.9	0.812
5	904.863	2872.45	0.798
6	750.219	2849.51	0.792
7	654.644	2835.86	0.788
8	595.575	2827.79	0.785
9	559.069	2822.96	0.784
10	536.506	2820.9	0.784
11	522.562	2819.34	0.783
12	513.944	2818.03	0.783
13	508.618	2817.32	0.783
14	505.285	2817.11	0.783



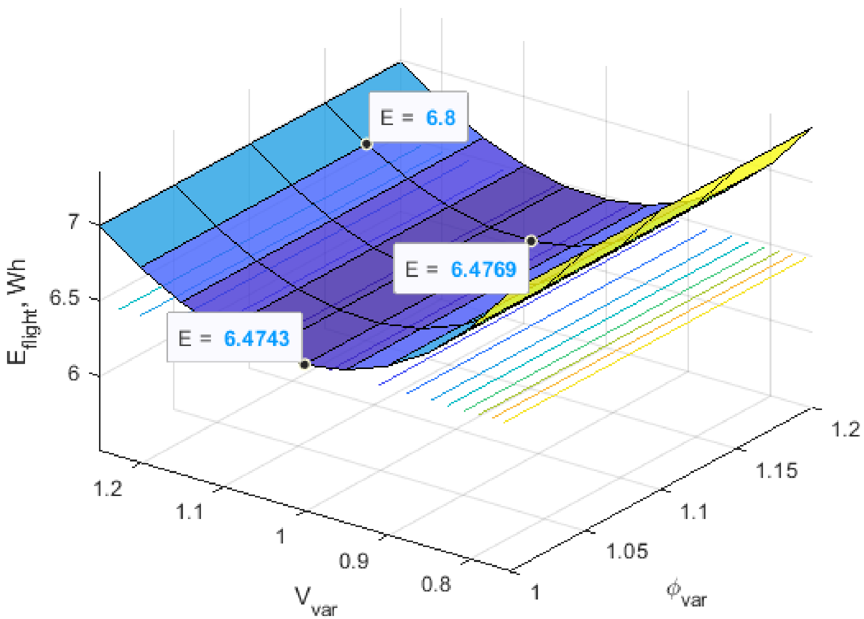

```
Wind scale = 1000.0 , Hcmd = 2215.02739051m
   Iteration     Max residual     Total nodes     Nodes added
       1          5.81e-01            10              18
...
       7          4.10e-13          6562            (5920)
Wind scale = 100.0 , Hcmd = 2215.02739051m , P = [ 0.92444636]
   Iteration     Max residual     Total nodes     Nodes added
...
       3          1.79e-02            82             (162)
Wind scale = 10.0 , Hcmd = 2215.02739051m , P = [ 0.92027459]
   Iteration     Max residual     Total nodes     Nodes added
...
       3          1.44e-02            82             (162)
Wind scale = 1.0 , Hcmd = 2215.02739051m , P = [ 0.89845575]
   Iteration     Max residual     Total nodes     Nodes added
...
       3          9.27e-03            82             (162)
Number of nodes is exceeded after iteration 3, maximum relative residual 9.27e-03.
...
// Local search for best height
 Func-count       x            f(x)          Procedure
    1          2215.03       3148.08         initial
    2          3274.97       3396.22         golden
    3          1559.95       3003.83         golden
    4          1155.08        2921.9         golden
    5          904.863       2872.45         golden
    6          750.219       2849.51         golden
    7          654.644       2835.86         golden
    8          595.575       2827.79         golden
    9          559.069       2822.96         golden
   10          536.506        2820.9         golden
   11          522.562       2819.34         golden
   12          513.944       2818.03         golden
   13          508.618       2817.32         golden
   14          505.285       2817.11         golden

Optimization terminated successfully;
The returned value satisfies the termination criteria
(using xtol = 10.0 )
```

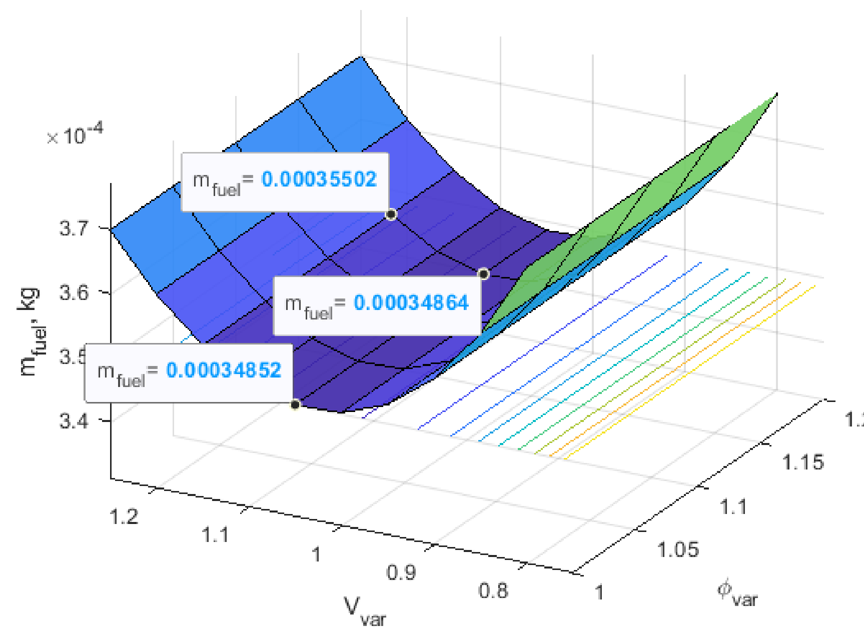
height

Fuel metric

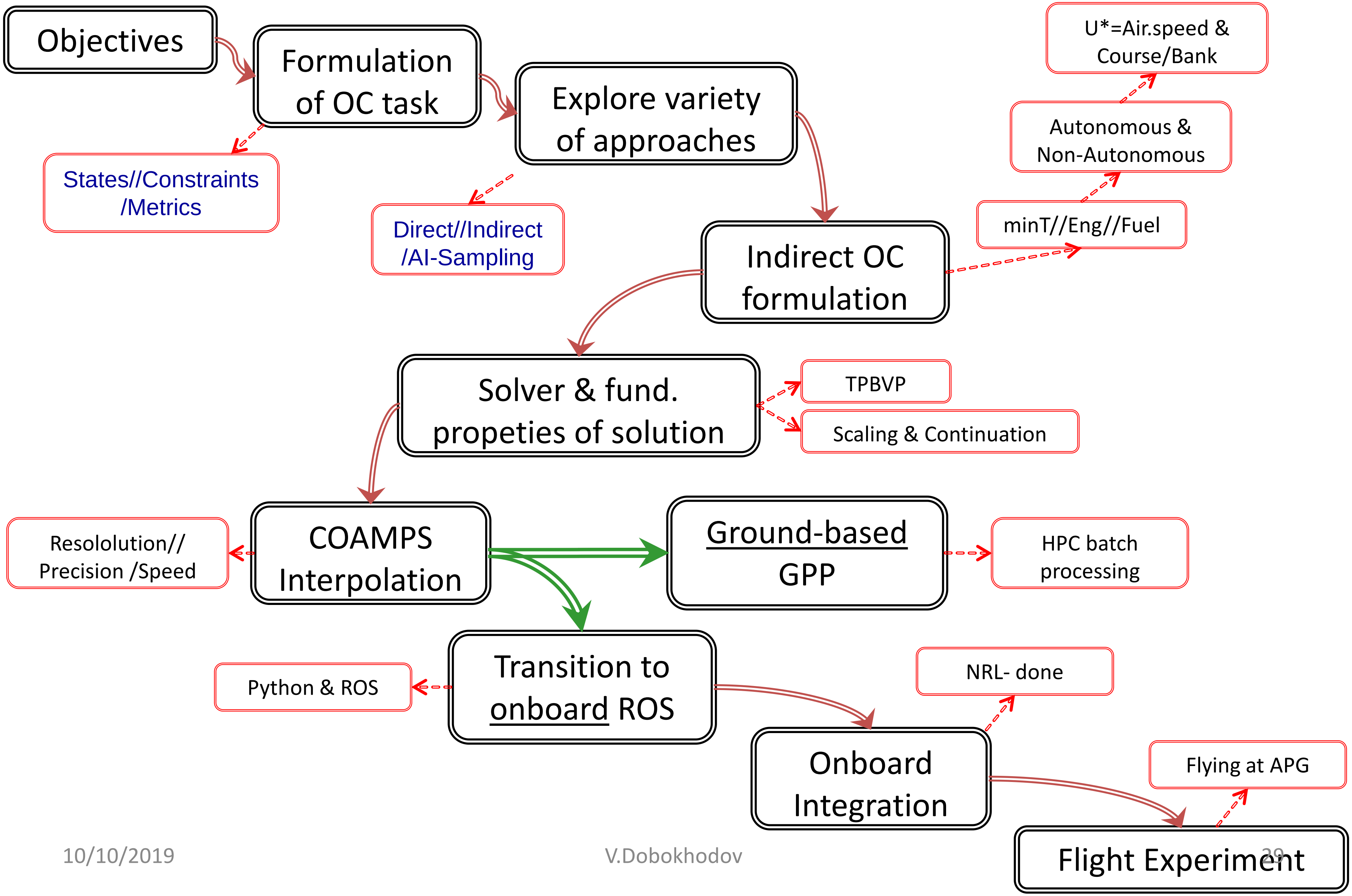
It takes 8.25 min on i7(~times 5 in Odroid)
Can we afford or Do we want this higher precision for the price of longer CPU time?

Synthetic Wind :: Hessian





Road Map


Objectives
Formulation of OC task
States//Constraints /Metrics
Explore variety of approaches
Direct//Indirect /AI-Sampling
U*=Air.speed & Course/Bank
Autonomous & Non-Autonomous
minT//Eng//Fuel
Indirect OC formulation
Solver & fund. propeties of solution
TPBVP
Scaling & Continuation
Resolol ution// Precision /Speed
COAMPS Interpolation
Ground-based GPP
HPC batch processing
Python & ROS
Transition to onboard ROS
NRL- done
Onboard Integration
Flying at APG
Flight Experiment