Developments in Computational Approaches for Model Predictive Control

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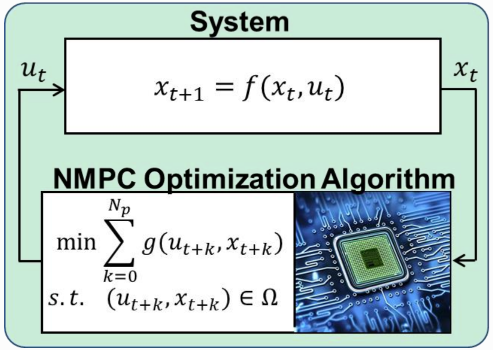

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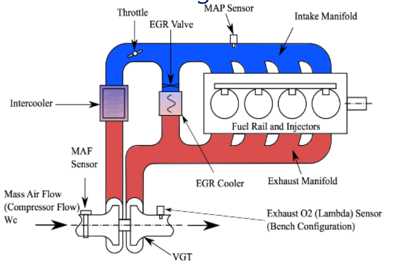
Research into Enhanced Numerical Methods for MPC

Objective: Improve methods for reducing computational cost of reliably solving Nonlinear Model Predictive Control (NMPC) problems

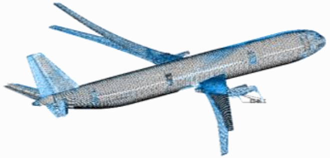
- Newton-Kantorovich inexact type methods for constrained NMPC
- Sensitivity-based warm starting strategies
- Semi-smooth predictor-corrector numerical strategies
- Approximating optimal finite horizon feedback by NMPC
- Characterizing closed-loop properties under NMPC involving inexact optimization
- Dynamically embedded MPC (DMPC)
- Software and improvements to semi-smooth quadratic programming algorithms for MPC

Diesel Engine Control

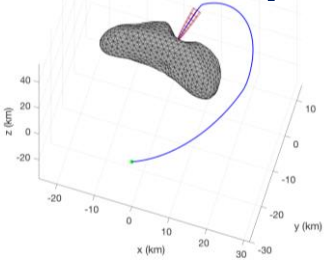


Autonomous Vehicle Control

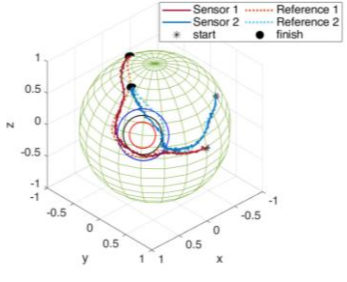
Maneuver Load Alleviation



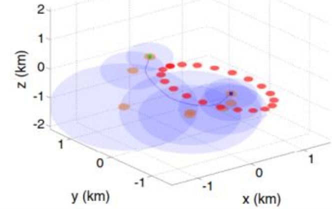
Asteroid Landing



Agile Satellite Attitude control



Debris Avoidance



Newton and Newton-Kantorovich Methods

Nonlinear equation:

$$f(x) = 0,\ \ f : \mathbb{R}^n \to \mathbb{R}^n,\ \ f \in C^1$$

Newton's method:

$$f(x^k) + \nabla f(x^k)(x^{k+1} - x^k) = 0$$

Inexact Newton-Kantorovich method:

$$\|f(x^k) + \nabla f(x^0)(x^{k+1} - x^k)\| \leq \zeta^k \|f(x^k)\|$$

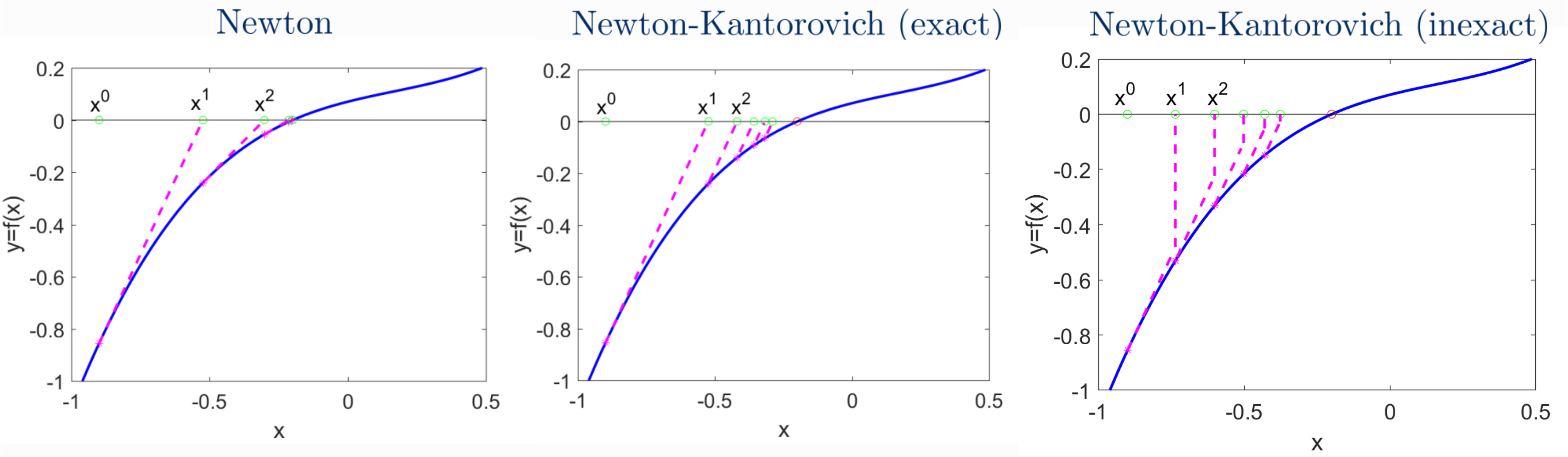
$$\zeta^k > 0,\ \ \zeta^k \to 0 \text{ as } k \to \infty$$

Computational cost reduction:

- $\nabla f(x^0)$ is computed only once and does not have to be updated at each iteration
- The use of the bound $\zeta^k \|f(x^k\|$ implies each iteration can be solved up to a certain tolerance

Newton and Newton-Kantorovich Methods

Example: $f(x) = 0.6050x^3 - 0.3004x^2 + 0.2692x + 0.0707$



$$\zeta^k = \left(\frac{3}{4}\right)^{k+1}$$

Newton-Kantorovich Methods for Constrained NMPC

Optimal control problem in MPC

$$\text{Minimize} \quad J(x,u) = \sum_{i=0}^{N-1} \ell(x_i,u_i) + \Phi(x_N)$$

subject to the constraints

$$x_{i+1} = f(x_i,u_i), \quad i = 0,1,\ldots,N-1, \quad x_0 \text{ given},$$

$$x_i \in \mathbb{R}^n, \quad u_i \in U_i, \quad i = 0,1,\ldots,N-1,$$

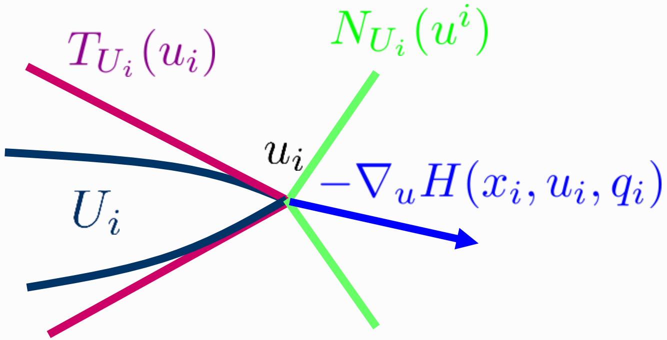
First order necessary optimality conditions:

$$x_{i+1} = f(x_i,u_i), \quad x_0 \text{ given},$$

$$q_{i-1} = -\nabla_x H(x_i,u_i,q_i), \quad q_{N-1} = -\nabla\Phi(x_N),$$

$$0 \in \nabla_u H(x_i,u_i,q_i) + N_{U_i}(u_i), \; i = 0,\ldots,N-1$$

$$H(x,u,q) = l(x,u) - \langle q, f(x,u)\rangle$$



Newton-Kantorovich Methods for Variational Inequalities

Variational inequality: $\varphi(x) + N_C(x) \ni 0, \quad C$ is closed and convex

$$N_C(x) = \begin{cases} \{y \in \mathbb{R}^n | \langle y, v - x \rangle \leq 0 \text{ for all } v \in C\} & \text{if } x \in C \\ \emptyset & \text{otherwise} \end{cases}$$

Inexact Newton-Kantorovich method:

$$\left(\varphi(x^k) + (A + B^k)(x^{k+1} - x^k) + N_C(x^{k+1})\right) \bigcap \mathbb{B}_{\zeta^k \sigma(x^k)}(0) \neq \emptyset$$

$\zeta^k > 0, \ \zeta^k \to 0$ as $k \to \infty$; $\sigma : \mathbb{R}^n \to \mathbb{R}^n$ loc. Lipschitz

$A = 0, B^k = \nabla\varphi(x^k), \sigma = 0, C = \mathbb{R}^n \Rightarrow$ standard Newton for equations

$A = \nabla\varphi(x^0), B^k = 0, \sigma = 0, C = \mathbb{R}^n \Rightarrow$ Newton Kantorovich for equations

$A = 0, B^k = \nabla\varphi(x^k), \sigma(x) = \|f(x)\|, C = \mathbb{R}^n \Rightarrow$ inexact Newton for equations

[1]Dontchev, Huang, **K.**, and Nicotra, "Inexact Newton-Kantorovich Methods for Constrained Nonlinear Model Predictive Control," *IEEE Transactions on Automatic Control*, **64**(9), 3602-3615, 2019.

Newton-Kantorovich Methods for Constrained NMPC

Under suitable assumptions, inexact Newton-Kantorovich iterations exhibit[1]

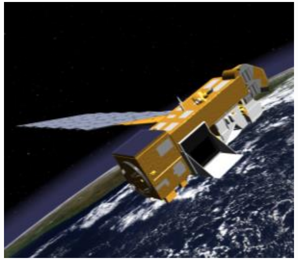
- r-linear convergence: $\|x^k - \bar{x}\| \leq aq^k$, $0 \leq q < 1$
- estimate for region of attraction is available

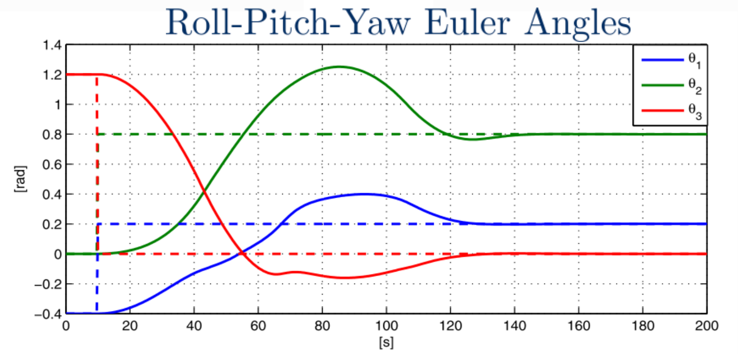
Applications to NMPC[1]:

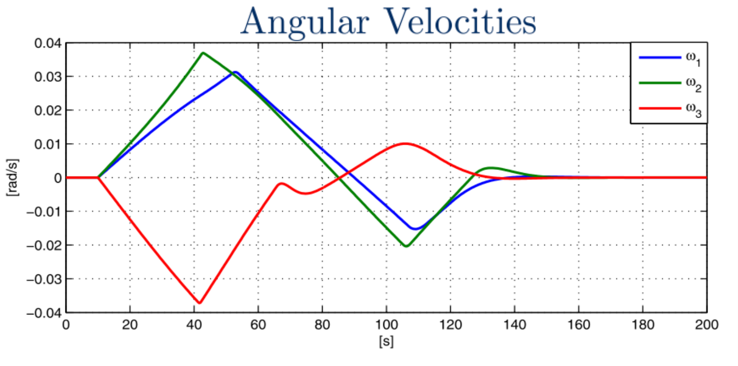
- Exact/Inexact SQP-Kantorovich Type Methods
- Nonsmooth Newton Type Methods

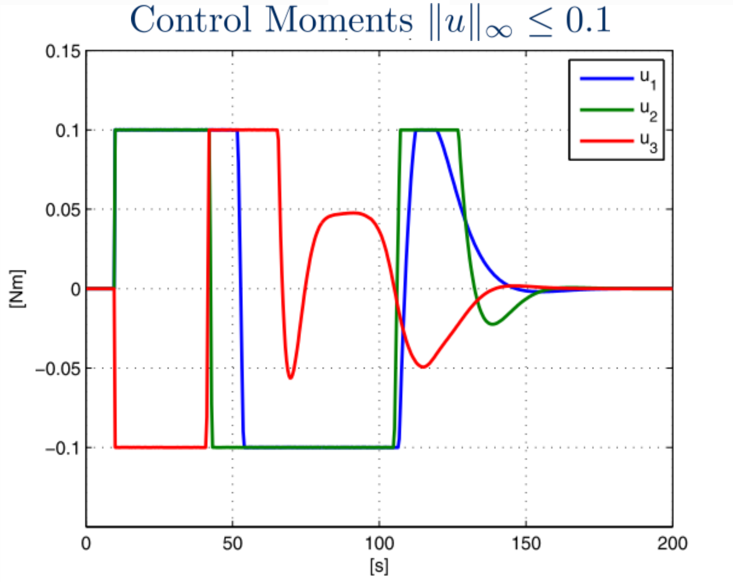
[1]Dontchev, Huang, **K.**, and Nicotra, "Inexact Newton-Kantorovich Methods for Constrained Nonlinear Model Predictive Control," *IEEE Transactions on Automatic Control*, **64**(9), 3602-3615, 2019.

Numerical Simulations: 3DOF Spacecraft Attitude Control





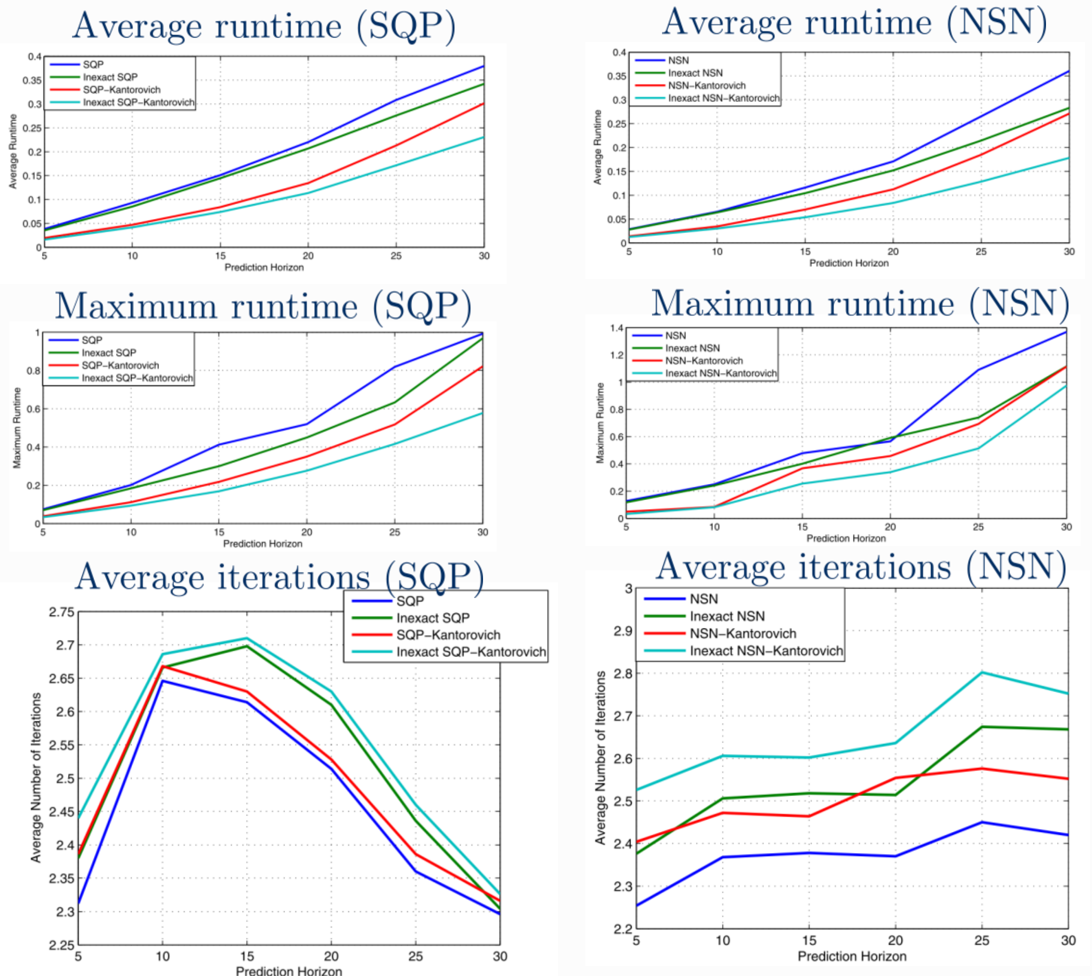




Numerical Simulations: Spacecraft Attitude Control

- First and second model derivatives computed numerically (center differences)

- Average and maximum runtime decrease even though average iterations increase



Sensitivity-Based Warm-Starting Techniques for MPC

- Exploit sensitivity estimates to initialize MPC optimization algorithm at next time step

Optimal control problem parameterized by initial state → Sensitivity analysis Predictor update → Warm-start solver Corrector update

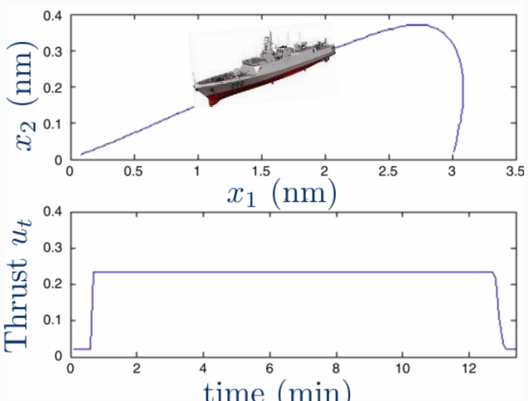
- Long history in MPC:

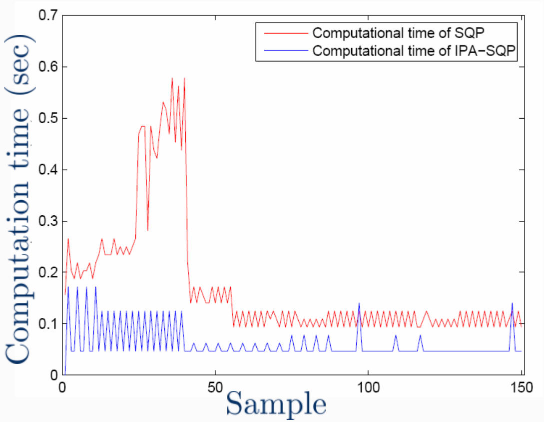
  Diehl (2001), Ohtsuka (2004), Zavala and Biegler (2009), Zavala and Anitescu(2010), Dinh et al., (2012), Jäschke et al. (2014), $\cdots$

Integrated Perturbation Analysis and SQP (IPA-SQP[1])

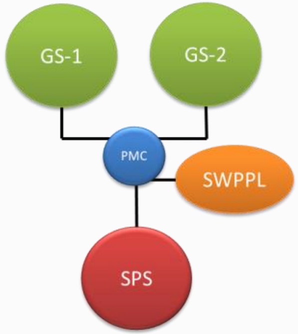
- State and control constraints considered
- Predictor updates defined based on Neighboring Extremal optimal control theory
- Integrated predictor-corrector updates
- Strategy to handle large perturbations

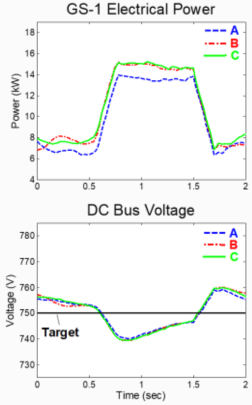
Ship Steering[1] (Nonlinear Model, 5 states, 140 steps horizon)





Ship Power Microgrid Management[2]





Sampling time		N = 5	N = 25	N = 50
20 ms	Opal-RT simulator	~ 1.5 ms	~ 6 ms	~ 12 ms

[1]R. Ghaemi, J. Sun, and K., "An Integrated Perturbation Analysis and Sequential Quadratic Programming Approach for Model Predictive Control," *Automatica*, **45**:2412-2418, 2009. [2]Park, et. al., IEEE TCST, 2015.

Predictor Based on Semiderivative of Solution Mapping

Parameterized discrete-time OCP

$$\min._{x,u} \quad J(x,u) = \varphi(x_N) + \sum_{i=0}^{N-1} \ell(x_i, u_i),$$

subject to

$$x_{i+1} = f(x_i, u_i), \quad i = 0, \ldots, N-1,$$
$$x_0 = p,$$
$$(x_i, u_i) \in Z_i, \ i = 1, \ldots, N-1,$$
$$u_0 \in U_0, \ x_N \in X_N$$

Polyhedral constraints

$$Z_i = \{(x,u) \mid E_i \left[x_i^{\mathrm{T}} \ u_i^{\mathrm{T}}\right]^{\mathrm{T}} \le c_i\}$$
$$X_N = \{x \mid E_N x \le c_N\}, \ U_0 = \{u \mid E_0 u \le c_0\}$$

Parameterized optimization problem

$$\min_{v} \quad J(v),$$

subject to $g(p, v) = 0, \quad v \in V,$

$$v = (u_0, \ x_1, \ u_1, \ \ldots \ , u_{N-1}, x_N)$$
$$V = U_0 \times Z_1 \times \ldots \times Z_{N-1} \times X_N$$

First-order necessary conditions

$$\mathcal{L}(p, v, q) = J(v) + q^{\mathrm{T}} g(p, v)$$
$$\nabla_v \mathcal{L}(p, v, q) + N_V(v) \ni 0,$$
$$g(p, v) = 0$$

Predictor Based on Semiderivative of Solution Mapping

- Semiderivative of solution mapping can be computed by solving a linear variational inequality, reduces to a QP
- Constraint qualification, such as LICQ, is not required
- Predictor QP has significantly fewer constraints, this can lead to reduced computational times for the overall predictor-corrector
- Previously active constraints can be deactivated during prediction step

Inputs: Δp, p_k, v_k, q_k

Predictor QP

$$\min_{\Delta v} \quad \frac{1}{2}\Delta v^T R_k \Delta v + (P_k \Delta p)^T \Delta v,$$

subject to

$$G_k \Delta v + Q_k \Delta p = 0$$
$$M_i \Delta v \leq 0, \;\; i \in \bar{A}$$
$$M_i \Delta v = 0, \;\; i \in A(\bar{v}) \setminus \bar{A}$$
$$\nabla_v \mathcal{L}(p_k, v_k, q_k)^{\mathrm{T}} \Delta v = 0$$
$$z = (v, q)\;, \;\; V = \{v \mid Mv \leq h\}$$

$$v_{k+1}^{-} \leftarrow v_k + \Delta v$$
$$q_{k+1}^{-} \leftarrow q_k + \Delta q$$

Corrector SQP iterations

$$z_{k+1} \in S(p_{k+1})$$

D. Liao-McPherson, M. Nicotra, A. Dontchev, K., and V. Veliov, ``Sensitivity-based Warm-starting for Model Predictive Control with Polyhedral State and Control Constraints," *IEEE TAC*, provisionally accepted, 2019, https://arxiv.org/abs/1906.11363

6DOF UAV Model Simulations

Model:

$$\dot{p} = v$$
$$m\dot{v} = TR(\theta)e_3 - mge_3$$
$$\dot{\theta} = R(\theta)\omega$$
$$J\dot{\omega} = -\omega \times (J\omega) + \tau$$

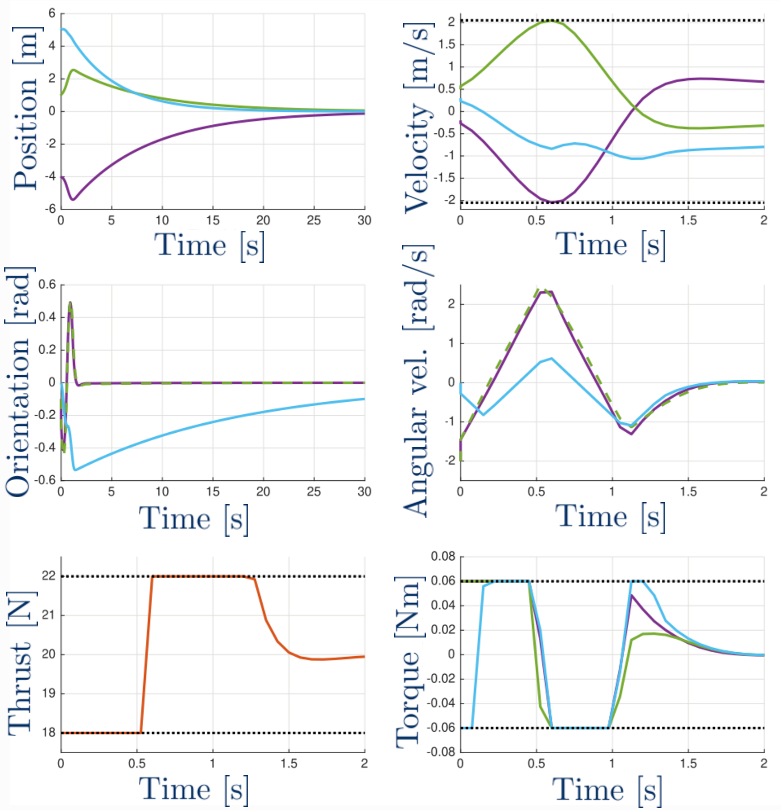
Cost Function:

$$\text{Minimize } x_N^{\mathrm{T}} P x_N + \sum_{k=0}^{N-1} x_k^{\mathrm{T}} Q x_k + u_k^{\mathrm{T}} R u_k$$

Constraints:

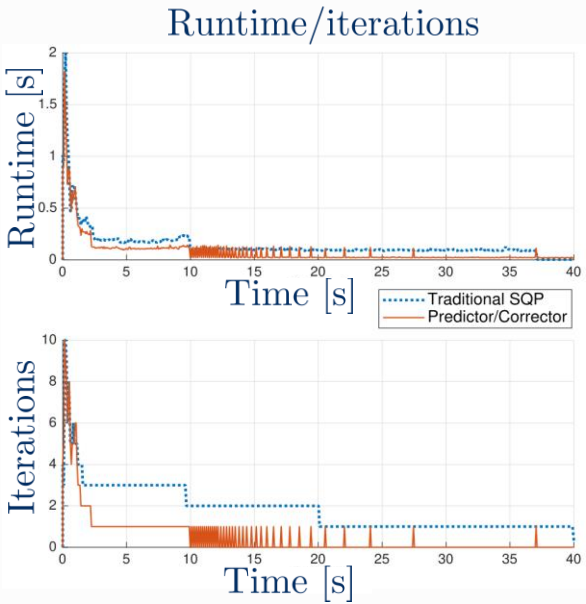
$$T \in [18, 22], \ \|\tau\|_\infty \leq 0.06, \ \|v\|_\infty \leq 2$$

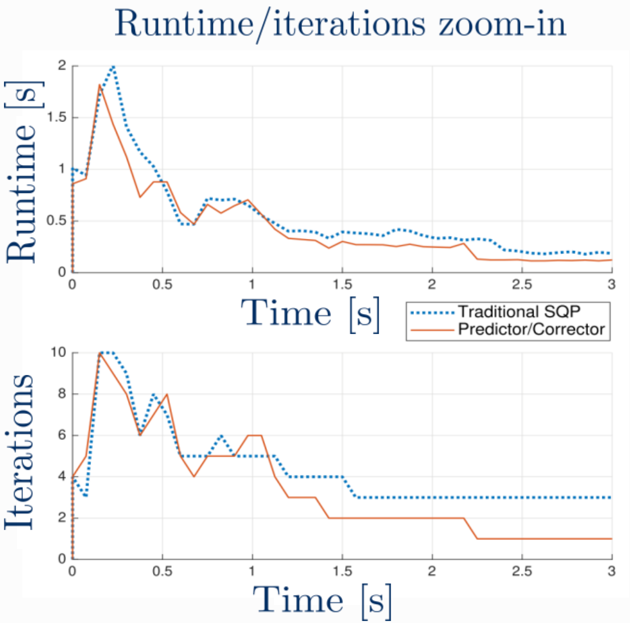
Closed-loop responses



6DOF UAV Model Simulations

- Comparing traditional SQP and Predictor-Corrector





- Reduced average runtime

The FBRS[1] Method for Convex Quadratic Programming

[1] FBRS = Fischer-Burmeister Regularized Smooth

Convex QP Problem

$$\min_x \frac{1}{2} x^{\mathrm{T}} H x + f^{\mathrm{T}} x$$
$$\text{s.t. } Ax \le b$$

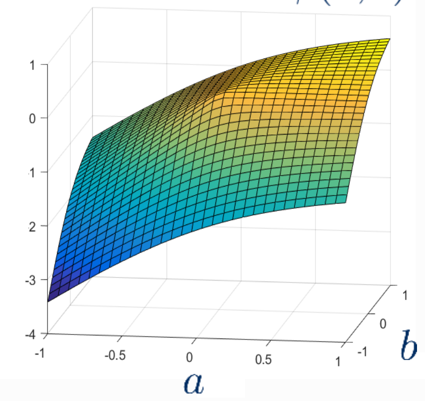
KKT Conditions

$$\nabla_x \mathcal{L} = Hx + f + A^{\mathrm{T}} v = 0$$
$$(b - Ax)^{\mathrm{T}} v = 0$$
$$v \ge 0, \quad b - Ax \ge 0$$

Fischer-Burmeister (FB) Function

$$\psi(a,b) = a + b - \sqrt{a^2 + b^2}$$
$$\psi(a,b) = 0 \Leftrightarrow a \ge 0, b \ge 0, ab = 0$$

FB Function $\psi(a,b)$



Rewrite the KKT conditions in terms of FB function

$$y = b - Ax, \quad z = (x, v)$$

$$F(z) = \begin{bmatrix} \nabla_x \mathcal{L}(x,v) \\ \psi(y_1, v_1) \\ \vdots \\ \psi(y_q, v_q) \end{bmatrix} = \begin{bmatrix} \nabla_x \mathcal{L}(x,v) \\ \phi(y,v) \end{bmatrix} = 0$$

The FBRS Method for Convex Quadratic Programming

Smoothing: Replace $F(z)$ by $F_\varepsilon(z)$

$$F_\varepsilon(z) = \begin{bmatrix} \nabla_x \mathcal{L}(x,v) \\ \phi_\varepsilon(y,v) \end{bmatrix}, \quad \phi_\varepsilon(y,v) = \begin{bmatrix} \psi_\varepsilon(y_1, v_1) \\ \vdots \\ \psi_\varepsilon(y_q, v_q) \end{bmatrix}$$

$$\psi_\varepsilon(a,b) = a + b - \sqrt{a^2 + b^2 + \varepsilon^2}$$

$F_\varepsilon(z)$ is smooth for $\varepsilon > 0$

Smoothed and regularized Newton iteration, line search

$$z_{k+1} = z_k - t_k K_k^{-1} F_{\varepsilon_k}(z_k)$$

$t_k \in [0,1]$ is line search step

$$K_k \in \partial F_{\varepsilon_k}(z_k) + \nabla R(z_k)$$

$R(z,\delta) = \begin{bmatrix} 0 \\ \delta(v+y) \end{bmatrix}$ is regularization function, $\delta > 0$

- $\partial F_{\varepsilon_k}(z_k)$ is Clark's generalized Jacobian (defined for $\varepsilon_k \geq 0$)
- $\varepsilon_k \to 0$ or ε_k is small
- δ is small

The FBRS[1] Method for Convex Quadratic Programming

Under reasonable assumptions[1], FBRS:

- Converges globally
- Exhibits asymptotic quadratic convergence rates
- Accepts arbitrary initial condition as a warm-start
- Is simple to implement
- Can exploit structure/sparsity
- No ill-condition for small ε

[1] D. Liao-McPherson, M. Huang, and K., ``A Regularized and Smoothed Fischer-Burmeister Method for Quadratic Programming with Applications to Model Predictive Control," *IEEE TAC* **64**(7): 2937-2944, 2019.

The FBRS[1] Method for Convex Quadratic Programming

$$\min_{x_1,x_2} \quad (x_1-1)^2 + (x_2-2.5)^2$$

$$\begin{bmatrix} 1 & -3 \\ 1 & 2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} 1 \\ 6 \\ 2 \end{bmatrix}$$

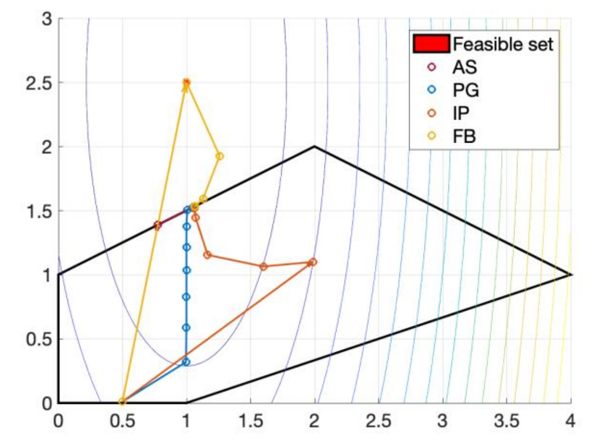
$$x_1 \geq 0, \; x_2 \geq 0$$

$$x^0 = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$$

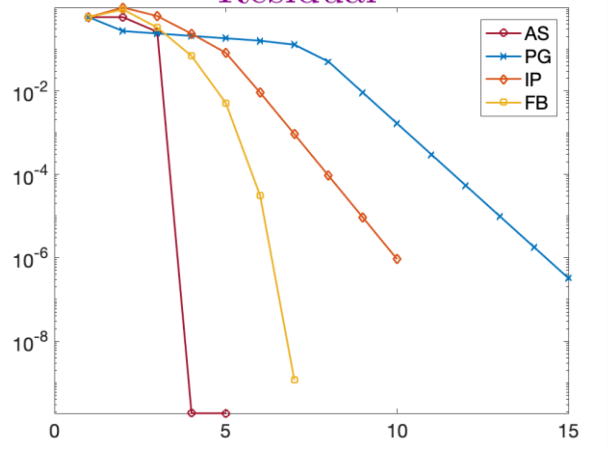
Residual:

$$\|x - \Pi_C\left[x - \alpha(Hx+f)\right]\|$$

$$\alpha = \frac{1}{\|H\|_F}, \quad C = \{x \mid Ax \leq b\}$$



Residual



Numerical Experiments[1]

TABLE I
SUMMARY OF PROBLEM SIZES.

	Asteroid	Diesel	F16	S/C
Number of States	6	5	5	6
Number of Controls	3	2	2	3
Horizon Length	70	10	N/A	10
Number of variables	280	30	12	31
Number of slacks	70	0	0	1
Number of constraints	490	70	1010	181
Number of QPs in sequence	300	3000	599	50

TABLE II
AVERAGE EXECUTION TIME FOR EACH SEQUENCE OF QPS. ENTRIES IN EACH COLUMN HAVE BEEN NORMALIZED BY THE FIRST ELEMENT.

	Asteroid	Diesel	F16	S/C
Normalization [msec]	17.75	0.11	2.14	0.18
FBRS	1	1	1	1
PDIP	4.06	9.62	0.45	7.40
QPKWIK	4.28	1.12	0.08	0.54
GPAD	N/A	17.94	10.97	0.81
Quadprog IP	3.60	75.04	16.23	121.77
ECOS	9.01	23.99	8.54	54.87

TABLE III
MAXIMUM EXECUTION TIME FOR EACH SEQUENCE OF QPS. ENTRIES IN EACH COLUMN HAVE BEEN NORMALIZED BY THE FIRST ELEMENT.

	Asteroid	Diesel	F16	S/C
Normalization [msec]	164.67	1.16	11.40	0.26
FBRS	1	1	1	1
PDIP	0.94	2.26	0.18	5.56
QPKWIK	3.24	0.73	0.06	0.91
GPAD	N/A	63.13	8.80	1.06
Quadprog IP	0.96	8.22	5.77	112.42
ECOS	1.31	3.95	2.40	44.82

Overall, FBRS appears competitive with state of the art solvers

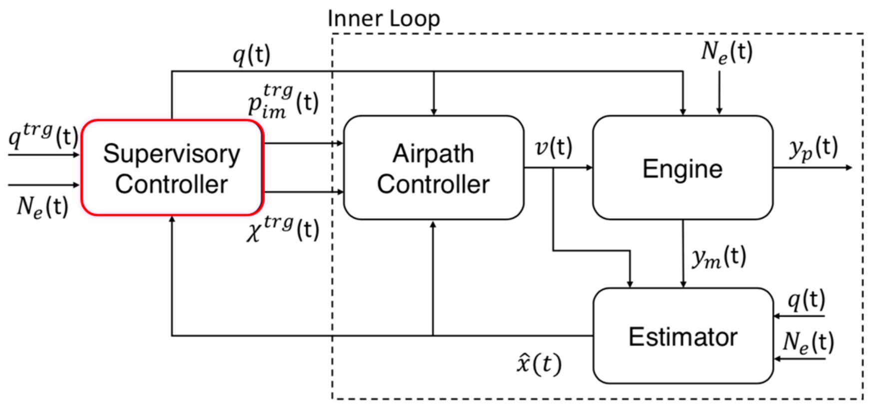
[1]D. Liao-McPherson, M. Huang, and K., ``A Regularized and Smoothed Fischer-Burmeister Method for Quadratic Programming with Applications to Model Predictive Control," *IEEE TAC* **64**(7): 2937-2944, 2019.

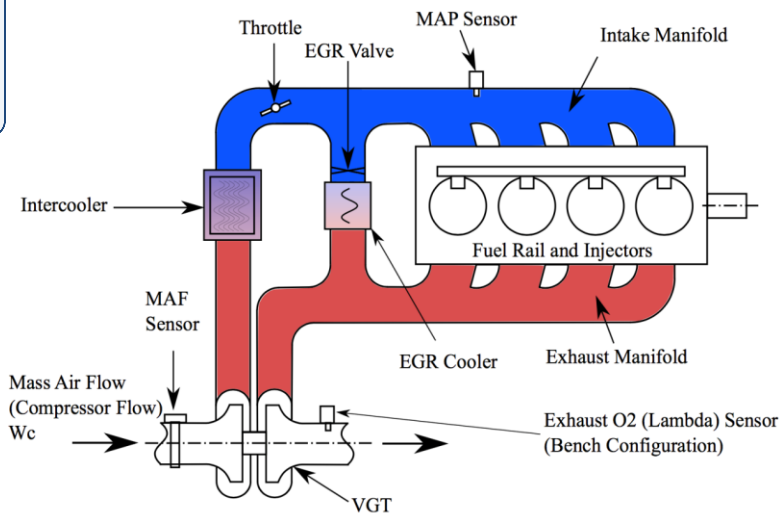
Application[1]: Diesel Engine Control

Supervisory NMPC controller to coordinate diesel engine fuel limiting and EGR rate set-point adjustment

The problem has:

- 5 states, 2 controls, 8 step horizon
- 17 optimization variables, 41 constraints

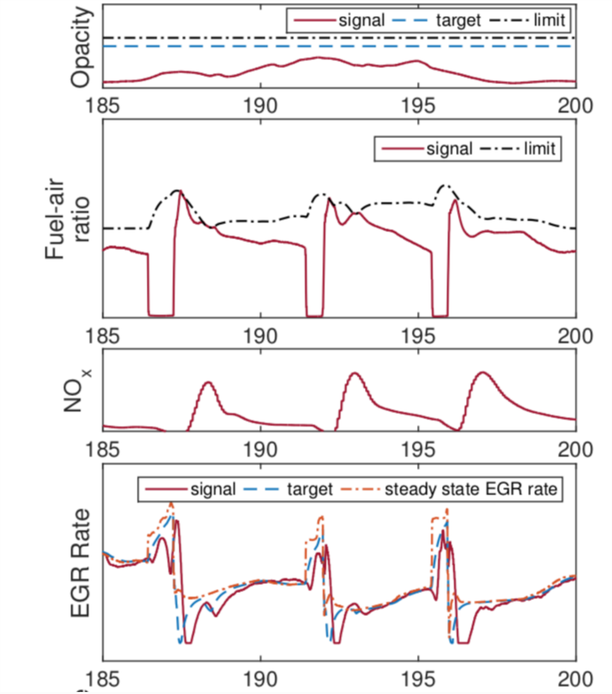


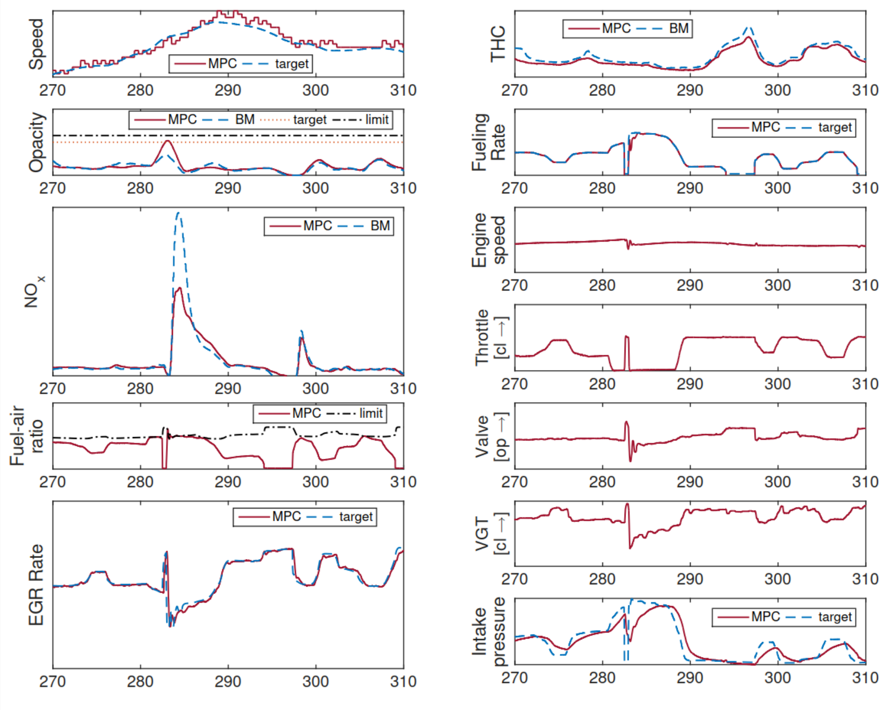


[1]D. Liao-McPherson, M. Huang, S. Kim, M. Shimada, K. Butts, and K., ``Model Predictive Emissions Control of a Diesel Engine Airpath: Design and Experimental Evaluation," *IJRNC, provisionally accepted*, 2019.

Diesel Engine Control: Experimental Results

- Implemented in DS1006 (2.6 GHz), experimentally tested on an engine
- Max. execution time $\leq 550\ \mu s$ per time step $\Rightarrow \approx 2$ kHz using FBRS
- Reduced NOx and HC emissions 10-15% without violation of smoke constraints





FBstab[1,2]: Proximally Stabilized Fischer-Burmeister Method

FBstab: Improved version of FBRS using proximal stabilization

[1]D. Liao-McPherson and **K.**, "FBstab: A Proximally Stabilized Semismooth Algorithm for Convex Quadratic Programming," https://arxiv.org/abs/1901.04046

[2]Matlab implementation: https://github.com/dliaomcp/fbstab-matlab

C++ implementation forthcoming later this month

Numerical Results: Dense Problems

FBstab version implemented in Matlab

- Uses dense built-in linear algebra
- Execution time measured on a rapid prototyping unit running a real-time OS
- Warm starting enabled

FBstab performance compared to

- Active set (KWIK)
- Regularized active set (NNLS)
- Previous version of FBstab (FBRS)
- Primal-dual interior point (PDIP)

FBstab outperforms other methods on larger spacecraft and copolymerization problems

	FBstab	FBRS	KWIK	NNLS	PDIP
Servo Motor, Normalization $= 1.11 \times 10^{-3}\ s$					
MAX	1.00	0.53	0.16	F	0.69
AVE	0.10	0.09	0.05	F	0.26
Spacecraft, Normalization $= 0.652\ s$					
MAX	1.00	F	F	1.33	1.06
AVE	0.21	F	F	0.46	0.65
Copolymerization, Normalization $= 1.06\ s$					
MAX	1.00	1.59	4.34	1.11	9.73
AVE	0.36	0.38	1.03	0.51	7.05

Normalized Speedgoat benchmarking reporting maximum and average QP solution times. All algorithms were warm-started.

Numerical Results: Sparse Problems

Implemented a prototype version of FBstab in Matlab to investigate scaling

- Copolymerization MPC problem with 18 states, 5 controls
- Horizon varied from 10 to 1000 steps

FBstab implemented with 4 different linear solvers

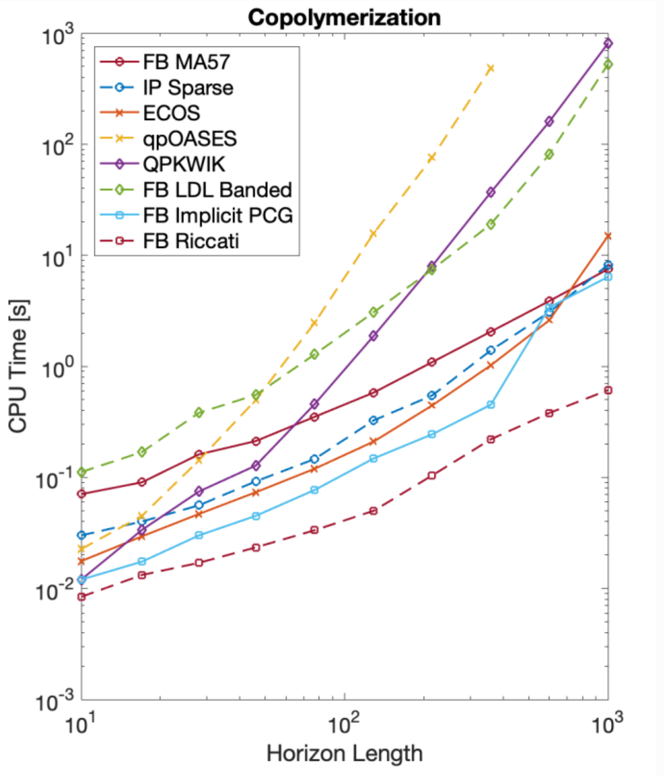
- State of the art sparse factorization (MA57)
- Banded LDL factorization
- Implicitly condensed conjugate gradient (PCG)
- Riccati like recursion

Benchmark: Quadprog (2017b), ECOS and QPKWIK (dual active set)

- Using sparse linear algebra (IP Sparse)
- Solving the condensed problem (IP Condensed)

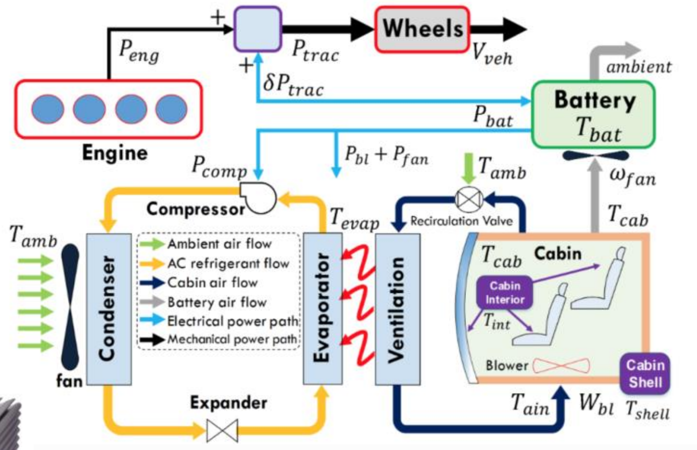
FBstab with the Riccati recursion solver is the quickest

- A power fit gives $O(N^{1.08})$ growth



Energy Efficient Climate Control (Eco-Cooling) in CAVs

- Exploit vehicle speed sensitivity of A/C efficiency
- Over 4% in vehicle energy savings in vehicle tests
- Speedgoat implementation of two layer MPC
- FBstab used as a part of SQP-based suboptimal MPC scheme

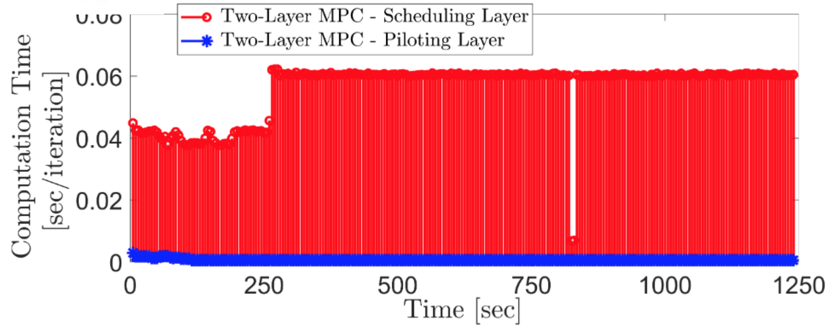


Scheduling layer

- 24 step horizon, 5 sec sampling time
- 2 states, 3 controls
- Average/Max execution time: 55/62 msec

Piloting layer

- 6 step horizon, 5 sec sampling time
- 2 states, 2 controls
- Average/Max execution time: 0.56/2.9 msec



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Semismooth Predictor-Corrector

Parameterized state and control-constrained OCP

$$\min_{w} f(w)$$
$$s.t.\ \ g(w,x) = 0$$
$$c(w) \leq 0$$
$$x = \text{given parameter}$$

KKT Necessary Conditions

$$\nabla_w L(w,\lambda,v,x) = 0$$
$$g(w,x) = 0$$
$$c(w) \leq 0, v \geq 0\,, c(w)^T v = 0$$

Lagrangian:

$$L(w,\lambda,v,x) = L(z,x) = \ f(w) + \lambda^T g(w,x) + v^T c(w)$$

Semismooth Predictor-Corrector

Root finding problem

$$F(z,x) = \begin{bmatrix} \nabla_w L(z,x) \\ g(w,p) \\ \psi(-c_1(w,x), v_1) \\ \vdots \\ \psi(-c_q(w,x), v_q) \end{bmatrix} = 0$$

Solution mapping:

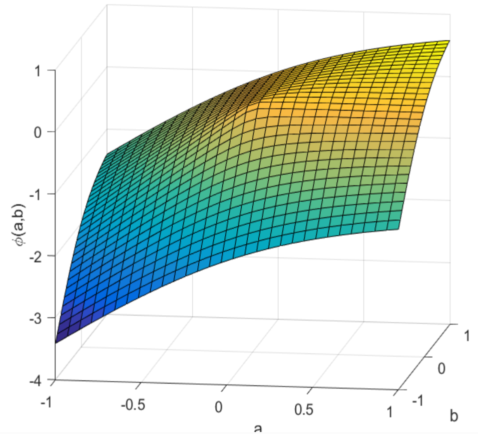
$$S(x) = \{z \mid F(z,x) = 0\}$$

Nonlinear complementarity function (NCP) $\psi: \mathbb{R}^2 \to \mathbb{R}$

$$\psi(a,b) = 0 \Leftrightarrow a \geq 0, b \geq 0, ab = 0$$

Example: Fischer-Burmeister function

$$\psi(a,b) = a + b - \sqrt{a^2 + b^2}$$



Semismooth Newton's Method

Clarke's Generalized Jacobian

How to solve $F(z,x) = 0$?

Semismooth Newton's Method

$$z_{i+1} = z_i - V_i^{-1} F(z_i, x), \ \ V_i \in \partial F(z_i, x)$$

This converges quadratically, i.e.,

$$\left|\left|z_{i+1} - S(x)\right|\right| \leq \eta \left|\left|z_i - S(x)\right|\right|^2$$

Fischer 1992, Qi et al. 1993

Semismooth Predictor

Directional derivative

What if x changes from $x \to x + \Delta x$?

Directional derivative predictor

$$z_{i+1} = z_i + S'(x; \Delta x)$$

Since $S(x) = \{z \mid F(z, x) = 0\}$

$$S'(x; \Delta x) = -V^{-1} B \Delta x,$$

$$V \in \partial_z F(S(x), x), \qquad B \in \partial_x F(S(x), x))$$

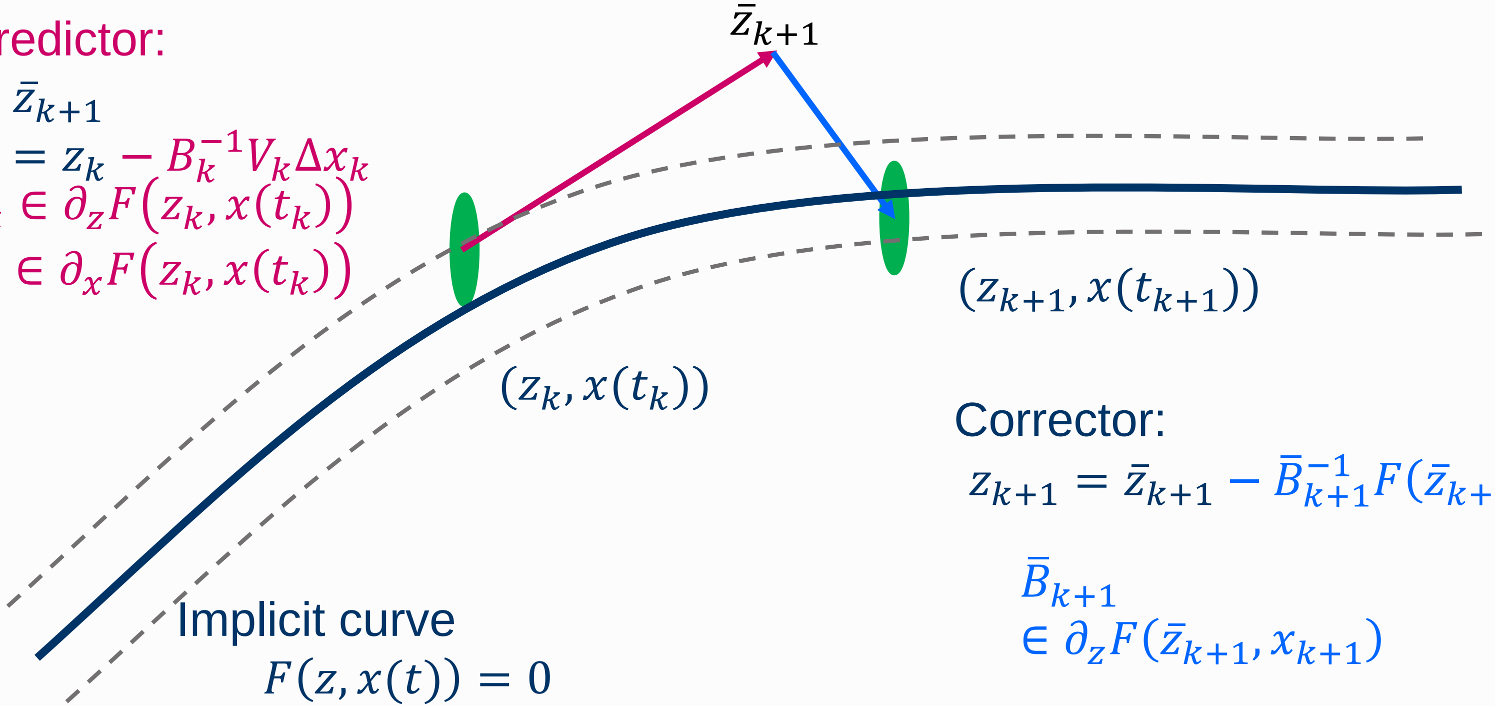
Semismooth Predictor-Corrector[1,2]

Predictor:

$$\bar{z}_{k+1} = z_k - B_k^{-1} V_k \Delta x_k$$

$$B_k \in \partial_z F\big(z_k, x(t_k)\big)$$

$$V_k \in \partial_x F\big(z_k, x(t_k)\big)$$



Corrector:

$$z_{k+1} = \bar{z}_{k+1} - \bar{B}_{k+1}^{-1} F(\bar{z}_{k+1}, x_{k+1})$$

$$\bar{B}_{k+1} \in \partial_z F(\bar{z}_{k+1}, x_{k+1})$$

[1]D. Liao-McPherson, M. Nicotra, and **K.**, "A semismooth Predictor Corrector Method for Suboptimal Model Predictive Control," *Proc. of ECC*, 2019; [2]D. Liao-McPherson, M. Nicotra, and **K.**, "A Semismooth Predictor-Corrector Method for Real-Time Constrained Parametric Optimization with Applications in Model Predictive Control," Proc. of CDC, 2018.

Semismooth Predictor-Corrector

Define:

$$e_k = z_k - S(x_k),\ \ \Delta x_k = x_k - x_{k-1}$$

Error bounds[1]:

$$||\bar{e}_k|| \leq \left||e_{k-1}|\right| + c\left||\Delta x_k|\right|^2$$

$$\left||e_k|\right| \leq \eta\left||\bar{e}_k|\right|^2$$

Provided

- $\Delta x_k, e_{k-1}$ are small enough
- $S(x)$ is locally Lipschitz

- Quadratic convergence rate
- Updates require linear system solutions only
- Matrices are guaranteed to be invertible near regular solutions

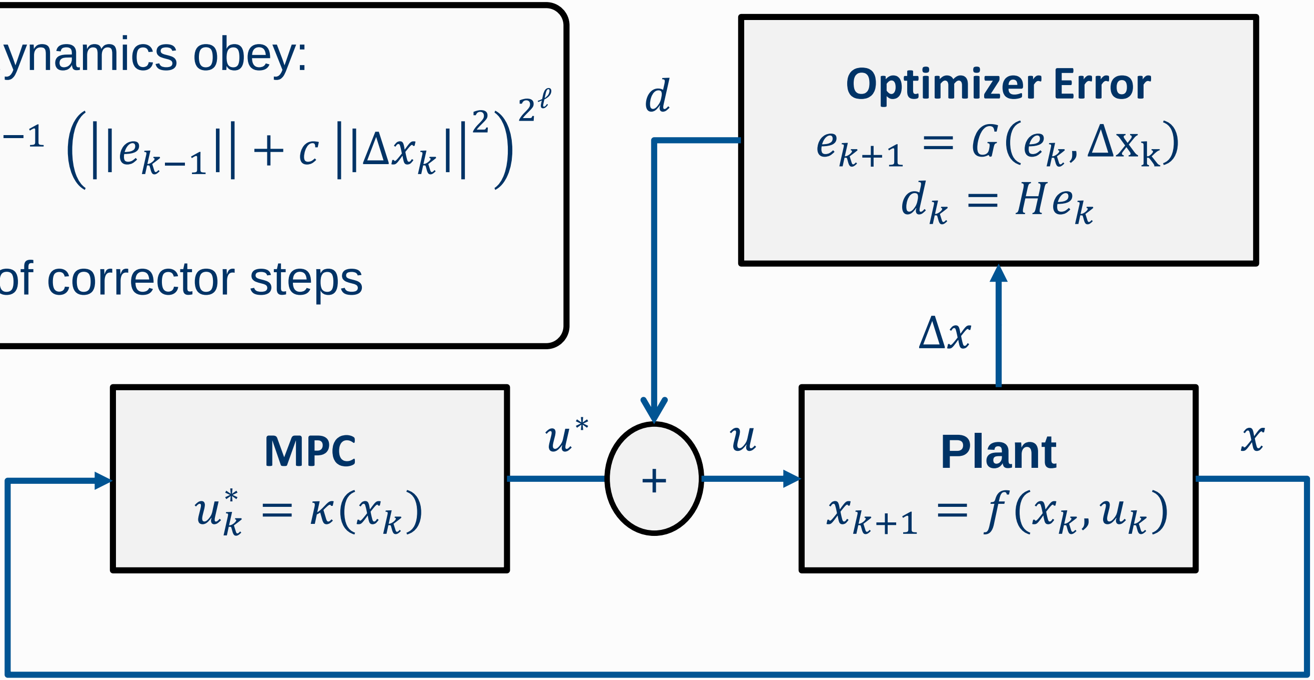
[1]D. Liao-McPherson, M. Nicotra, and **K.**, "A Semismooth Predictor-Corrector Method for Real-Time Constrained Parametric Optimization with Applications in Model Predictive Control," Proc. of CDC, 2018.

Analysis of Suboptimal MPC with Fixed Number of Iterations

The error dynamics obey:

$$||e_k|| \leq \eta^{2^{\ell}-1} \left(||e_{k-1}|| + c \, ||\Delta x_k||^2 \right)^{2^{\ell}}$$

ℓ: number of corrector steps



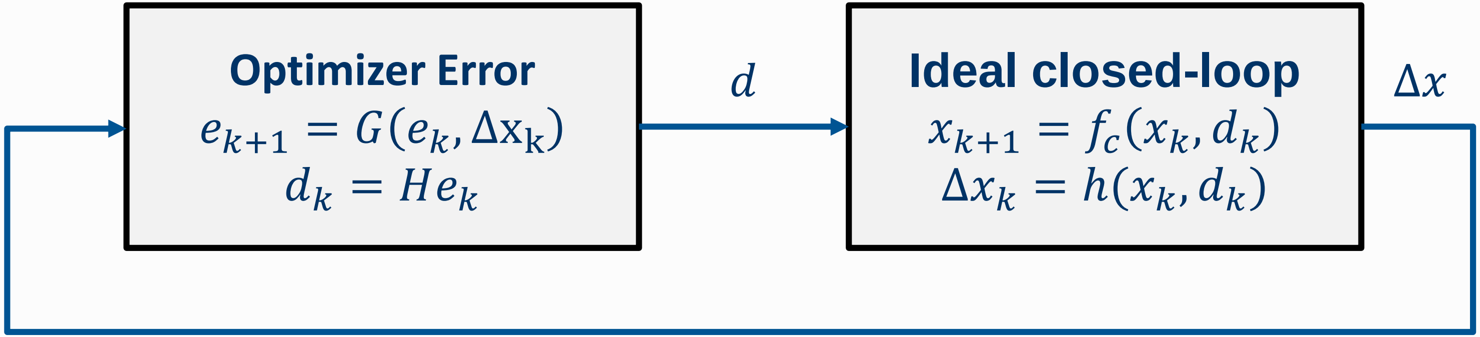
Analysis of Suboptimal MPC with Fixed Number of Iterations

$$||d||_\infty \le \gamma_2\left(||\Delta x||_\infty, \ell\right)$$

$\gamma_2 \in \mathcal{KL}$
Monotonically decreasing in ℓ

$$||\Delta x||_\infty \le \gamma_1\left(||d||_\infty\right)$$

$\gamma_1 \in \mathcal{K}$
Limon et al. 2009



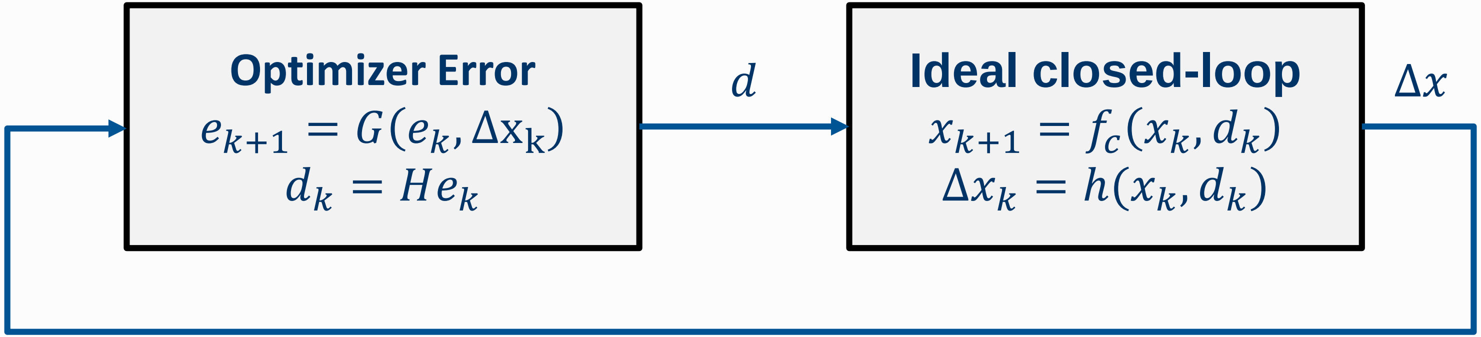
$$f_c(x, d) = f(x, \kappa(x) + d)$$

$$h = f(x, d) - x$$

Analysis of Suboptimal MPC with Fixed Number of Iterations

System is asymptotically stable if

$$\gamma_1 \circ \gamma_2(s, \ell) \leq s, \qquad \forall s \geq 0$$



$$f_c(x, d) = f(x, \kappa(x) + d) \qquad\qquad h = f(x, d) - x$$

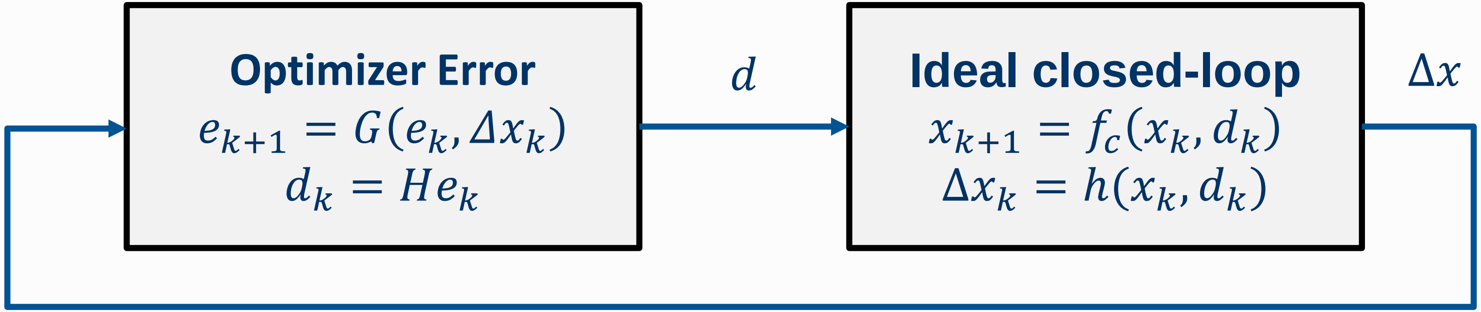
Asymptotic Stability with Suboptimal MPC

Consider the system below if:

- The system under the optimal MPC control law is LISS
- Solution mapping is Lipschitz continuous

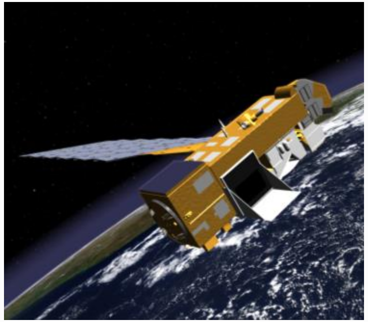
Then $\exists \ell^* < \infty$ such that if $\ell \geq \ell^*$ corrector iterations are performed

- The system is locally asymptotically stable



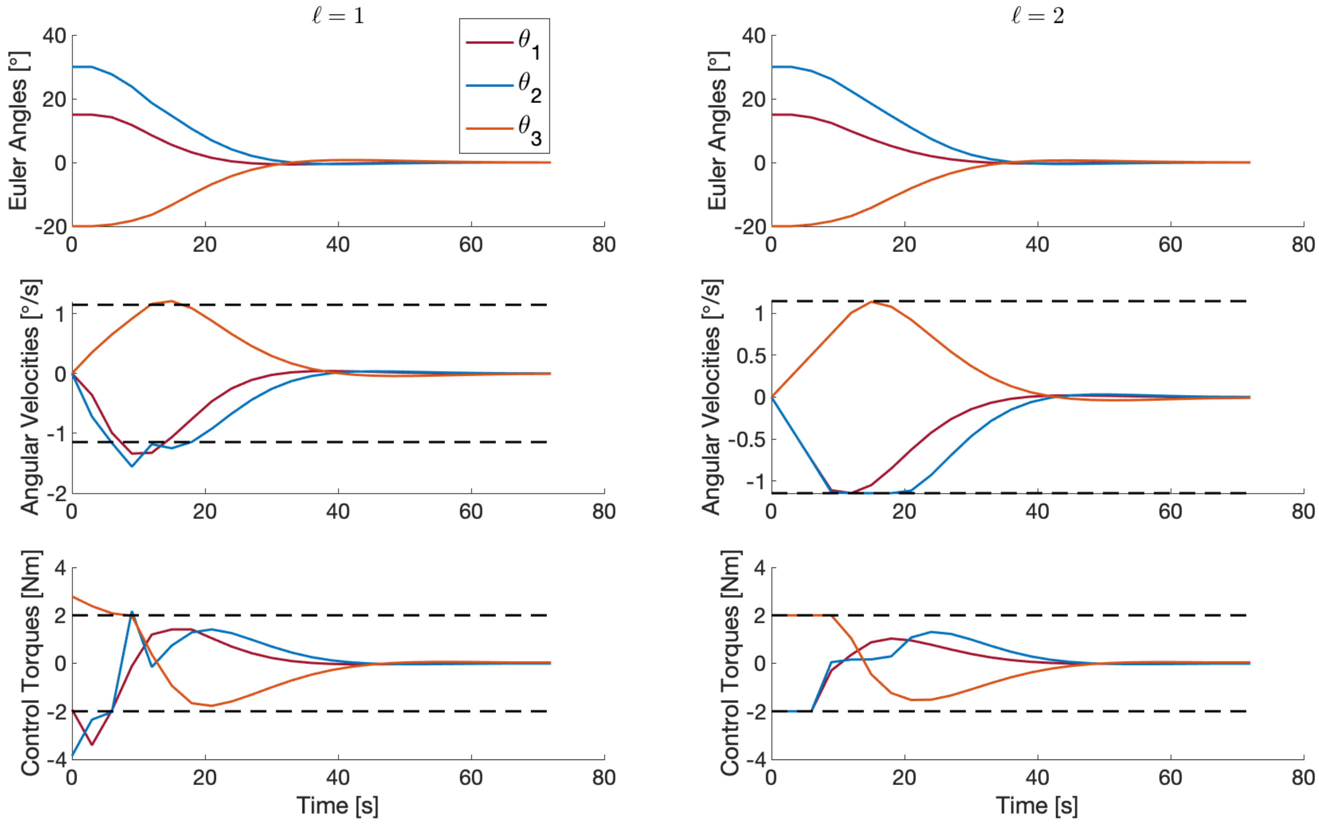
- A region $\Omega \times \mathcal{E}$ of the origin can be shown to exist such that if $\ell \geq \bar{\ell}$ and initial condition is in $\Omega \times \mathcal{E}$ then constraints will be satisfied.

3DOF Spacecraft Attitude Control: Numerical Simulations



Worst case execution times:

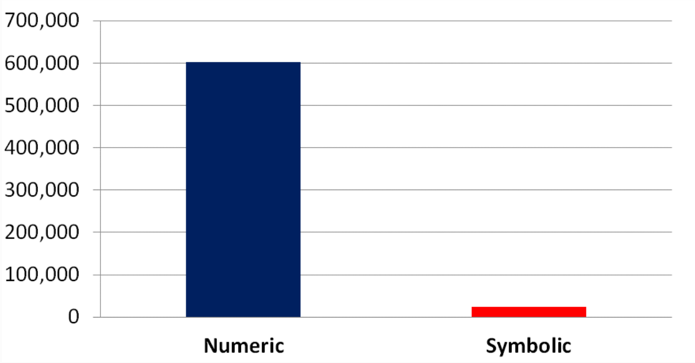
SSPC: 0.0371 s
fmincon: 1.41 s
IPOPT: 0.9701s



Symbolic code optimization

- Symbolic derivatives
- Extraction of common sub-expressions
- Elimination of multiplications by zeros
- Constant propagation
- Symbolic simplifications of linear equation solve
- Custom generated solver
- Symbolic control design environment (SCDE)

Solver's Impact on Performance
(Structured, Multiple shooting)
Fully Optimized Operation Count



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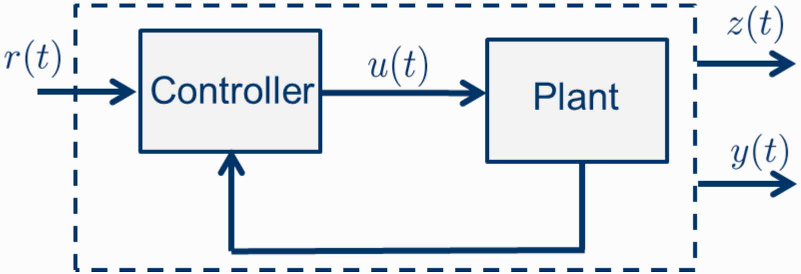
COLLEGE OF ENGINEERING
AEROSPACE ENGINEERING
UNIVERSITY OF MICHIGAN

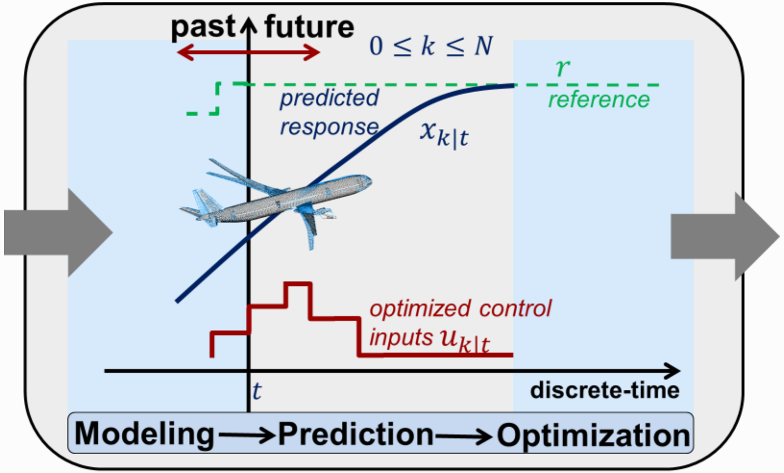
Developments in Computational Approaches for Model Predictive Control

Speaker: Ilya Kolmanovsky, Department of Aerospace Engineering, The University of Michigan

Abstract: Model Predictive Control (MPC) leads to algorithmically defined nonlinear feedback laws for systems with pointwise-in-time state and control constraints. These feedback laws are defined by solutions to appropriately posed dynamic optimization problems that are (typically) solved online. The talk will provide an overview of recent research by the presenter and his students and collaborators into several computational strategies for MPC. These strategies include Newton-Kantorovich inexact methods, sensitivity-based warmstarting which exploits predicting changes to a parameterized optimal control problem based on semiderivative of the solution mapping, improvements to semismooth methods for solving convex quadratic programs, and semismooth predictor-correct methods for suboptimal MPC. Potential for implementation and impact on applications such as engine control, thermal management of cabin and battery in connected and automated vehicles, and spacecraft control will also be discussed.

Model Predictive Control (MPC)





$$F(x_{N|t} - x_e(r)) + \sum_{k=0}^{N-1} L(z_{k|t} - r, u_{k|t})$$

$$\rightarrow \min_{u_{0|t}, \cdots, u_{N-1|t}}$$

subject to

$$x_{k+1|t} = f(x_{k|t}, u_{k|t}),$$
$$x_{0|t} = x(t), \; r = r(t),$$
$$z_{k|t} = g(x_{k|t}),$$
$$y_{k|t} = \begin{bmatrix} u_{k|t} \\ h(x_{k|t}) \end{bmatrix} \in Y,$$
$$x_{N|t} \in X_f(r) \subset X$$

$$u(t) = u_{MPC}(x(t), r(t)) = u^*_{0|t}$$

MPC is a nonlinear feedback law defined by the first move of the optimal control sequence obtained as a solution to a constrained optimal control problem