Oral Presentations

Title: Dissipativity of subsystem interconnections and distributed economic model predictive control

Authors: Philipp N. Koehler and Frank Allgower

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Abstract:

A Dynamic Programming approach on a tree structure for finite horizon optimal control problems

Alessandro Alla, Department of Mathematics, PUC-Rio, Rio de Janeiro, Brazil <u>alla@mat.puc-rio.br</u>

Abstract: The classical Dynamic Programming (DP) approach to optimal control problems is based on the characterization of the value function as the unique viscosity solution of a Hamilton-Jacobi-Bellman (HJB) equation [2]. The DP scheme for the numerical approximation of viscosity solutions of those equations is typically based on a time discretization which is projected on a xed space triangulation of the numerical domain. The time discretization can be done by a one-step scheme for the dynamics and the projection on the grid typically uses a polynomial interpolation.

In this talk, we will discuss a new approach for nite horizon optimal control problems where we compute the value function on a tree structure built directly by the time discrete dynamics avoiding the use of a space triangulation to solve the HJB equation (see the recent work [1]). This allows to drop the cost of the space interpolation and the tree will guarantee a perfect matching with the discrete dynamics. We will also provide error estimates (see [5]) for the algorithm if the dynamics is discretized with an Euler method. Furthermore, this approach has been extended to high-order schemes and we will show some examples of second order approximation schemes. Finally, we will show the eectiveness of the method for the control of PDEs. This is a joint work with Maurizio Falcone (La Sapienza, Roma) and Luca Saluzzi (GSSI, L'Aquila).

Title: Nonlinear Control of Voluntary Immunosuppression Treatment Dynamics in Renal Transplant Recipients

Speaker H. T. Banks, Center for Research in Scientific Computation, North Carolina State University, Raleigh, NC, USA, <u>htbanks@ncsu.edu</u> Abstract:

Kidney transplant recipients are put on a lifelong regime of immunosuppressants to prevent the body from rejecting the allograft. Suppressing the immune system renders the body susceptible to infections. The key to a successful transplant is to ensure the immune system is sufficiently suppressed to prevent organ rejection but adequately strong to fight infections. Finding the optimal balance between over and under suppression of the immune response is crucial in preventing allograft failure. In this presentation we present a feedback control formulation to predict the optimal amount of immunosuppression required by renal transplant recipients in the context of infections caused by the human polyomavirus type 1, also called the BK Virus (BKV). We use Receding Horizon Control (RHC) methodology to construct the feedback control. Data as it is currently collected provides information for only some model states. Using an improved model for BK infection in renal transplant recipients which includes the RHC methodology, we implement Non-Linear Kalman Filtering to introduce feedback into the system. Extended Kalman Filtering (EKF) and Ensemble Kalman Filtering (EnKF) were compared for their ability to provide feedback for the non-linear controller when some states of the system are not observable. Combining RHC and either EKF or EnKF allows the design of the desired adaptable treatment strategies. We conclude that using the presented methodology, an individualized adaptive treatment schedule can be built for renal transplant recipients. This represents joint efforts with Neha Murad, Nicholas Myers and H.T. Tran.

Title: NLP Concepts and Problem Formulations for Robustly Stable, High Performance NMPC

Speaker: L. T. Biegler, Chemical Engineering Department, Carnegie Mellon University Pittsburgh, PA 15213, <u>biegler@cmu.edu</u>

Speaker: Olivier Bokanoski, Laboratoire Jacques Louis Lions, Université Paris Diderot (Paris 7) and Ensta-Paris Tech, <u>olivier.bokanowski@gmail.com</u>

Title: Neural Networks for the approximation of Hamilton-Jacobi equations related to deterministic optimal control problems

Abstract: We propose two schemes based on Deep Neural Networks for the approximation of time-dependent first order Hamilton-Jacobi equations related to deterministic optimal control problems.

Speaker: Francesco Borelli, UCB

Howard Penn Brown Professor, Department of Mechanical Engineering University of California, Berkeley

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Title: Nonlinear MPC for Iterative Tasks: Challenges and Opportunities

This talk will focus on model predictive control (MPC) for autonomous systems performing iterative tasks. These control design problems are common in autonomous and semi-autonomous systems. Applications include transportation, energy and manufacturing systems. I will first provide an overview of the theory and tools that we have developed for the systematic design of iterative learning predictive controllers. Then, I will focus on the computational challenges associated with the use of data to formulate and solve nonlinear MPC problems which autonomously improve performance in iterative tasks while guaranteeing safety. Throughout the talk I will focus on autonomous manipulators and cars to motivate our research and show the associated computational challenges.

More info on: www.mpc.berkeley.edu

Speaker: Jerome Darbon, Brown University, jerome_darbon@brown.edu Title: On Hamilton-Jacobi partial differential equations and architectures of neural networks

Abstract:

We propose new and original mathematical connections between some classes of Hamilton-Jacobi (HJ) partial differential equations (PDEs) with initial data and neural networks (NNs). Specifically, we show that the physics contained in some HJ PDEs can naturally be encoded by some specific classes of NN architectures. We provide a mathematical analysis that completely characterizes the capability of these architectures to exactly represent solutions of some HJ PDEs. These theoretical results naturally yield efficient NN-based numerical methods for evaluating solutions of some HJ PDEs in high dimension without using grids or numerical approximations. We also briefly present some preliminary theoretical and numerical results for learning solutions of inverse problems involving HJ PDEs that correspond to learning the Hamiltonian or the initial data or both from data.

Speaker: Vladimir Dobrokhodov, NPS vndobrok@nps.edu

Title: Onboard Trajectory Planning for a new Class of Hybrid Aircraft Abstract: The discussion presents computational challenges of implementing realistic trajectory optimization problem onboard of a new class of hybrid aircraft that is capable of harvesting environmental energy of wind, solar irradiance, and the hydrogen fuel. Despite the numerous advances in numerical methods together with the exponential increase in computational power including the GPU and parallel processing capabilities, the optimal routing for this new class of aircraft presents a task that is still hard to solve in close to real-time pace. The task is formulated as a classical two-point boundary value problem and is solved by applying Pontryagin maximum principle. Despite the analytically synthesized optimal controls the computational efficiency still represents a bottleneck for its numerical implementation onboard modern CPU/GPU. Thus the talk considers a number of classical techniques and some new ideas of homotopy that enable at least 100 times faster computational performance. The approach solves the problem of initial guess by designing a continuation algorithm that is based on physical insight into the flight dynamics. Not only it initializes the next step of continuation, but it also serves as a reference for the comparison of energy expenditures along the energy optimal and the shortest routes. The developed method is used to solve a practically valuable task of path planning of a long endurance flight of a UAV over x1000nm distance

Title: Basis adaptation for max-plus eigenvector methods.

Speaker: Peter M. Dower

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Abstract: Max-plus eigenvector methods, developed for solving specific classes of nonlinear continuous time finite dimensional optimal control problems, are founded on the attendant max-plus linearity, semiconvexity, and semigroup properties of dynamic programming. Together, these properties imply the existence of an iterative coordinate vector representation for the associated value function, defined with respect to a countably infinite basis for a corresponding vector space of semiconvex functions. This basis consists of quadratic functions defined on the underlying finite dimensional state space, whose locations form a dense subset. A finite cardinality truncation of this basis yields a projection of the value function onto a corresponding subspace of semiconvex functions, and it is this projection that provides the approximation returned by the max-plus eigenvector method. The extent to which the curse-of-dimensionality is attenuated in this type of method is largely determined by the basis truncation selected, and in particular, whether that truncation yields a sufficiently sparse value function approximation to within a required (truncation) approximation error bound. A priori, this selection is typically unknown, as it depends on the value function solution being sought. Recent efforts to address this limitation are exploiting the dependence of the Hamiltonian back-substitution error on the set of basis function locations in order to facilitate basis adaptation. In particular, given a finite cardinality set of basis functions, a standard max-plus eigenvalue vector method is applied to yield a value function approximation. This value function approximation is used to identify a tessellation of a Hamiltonian backsubstitution error level set, in which each convex polytope corresponds to the set of states for which a particular basis function is active, and the approximation error is within a coarse bound. Elements of this tessellation, and hence individual basis functions, are identified that correspond to worst-case errors, and new basis functions added in directions that refine these errors. Unused basis functions are pruned, and the basis is updated. The algorithm proceeds iteratively, with the max-plus eigenvector method applied at each step to the adapted basis as indicated. Issues concerning implementation, ease-of-use, and scalability are discussed via examples.

Title: Route Planning Problems and Hybrid Control

Speaker: Roberto Ferretti, U. Rome III, <u>ferretti@mat.uniroma3.it</u> Abstract: In its simplest formulation, the so-called "route planning problem" for sailing boats consists in minimizing the expected time to reach a given target for a vessel sailing in a partly stochastic wind field. A change of direction (especially when tacking) might be associated to a time loss, which is in fact a crucial point in short-course races. This transition cost makes it natural to formulate the problem in term of stochastic hybrid control.

The related numerical dynamic programming techniques have been studied recently for the case of deterministic systems, and extended by the authors to the specific framework of route planning. In this talk, we will discuss a detailed hybrid model to formulate the optimal route planning in the case of both fleet races and match races, and provide a convergent numerical approximation. We will also present numerical examples showing the good agreement between the proposed model and the heuristically known features of the optimal strategy.

Speaker: Rolf Findeisen, rolf.findeisen@ovgu.de

Title: Finite-Step Lyapunov Function Meet Predictive Control: Towards Efficient Distributed Control

Navid Noroozi and Rolf Findeisen

Laboratory for Systems Theory and Automatic Control

Otto von Guericke Universität Magdeburg

Abstract; Finite step Lyapunov functions are a generalization of standard Lyapunov functions. In comparison to standard approaches finite step Lyapunov theory only requires a decrease after a finite number of steps to ensure stability. In the first part of this talk we outline how the task of stabilization can be reformulated as an optimization problem involving finite step Lyapunov functions. Based on the reformulation we present efficient model predictive control schemes, that allow to guarantee stability. We furthermore outline how these formulations can be expanded towards distributed predictive control subject, which can tolerate loss of information between the subsystems.

Title: Solving HJB equations on adaptive sparse grids Speaker: Jochem Gacke, U. Bonn garcke@ins.uni-bonn.de Abstract:

We investigate sparse grid discretization as an approach to solve the HJB equation efficiently while addressing the curse of dimensionality to some extent. We propose to apply a semi-Lagrangian scheme using spatially adaptive sparse grids. Sparse grids allow the discretization of the value functions in (higher) space dimensions since the curse of dimensionality of full grid methods arises to a much smaller extent. For additional efficiency an adaptive grid refinement procedure is explored.

In view of the underlying approximation theory for HJB equations we investigate properties of sparse grid discretization regarding monotonicity, where several counter examples show that this property does not hold in general. On the other hand, we present several numerical examples in high dimensions and investigate the empirical convergence behavior, while studying the effect some of the parameters characterizing the sparse grid have on the accuracy of the value function or the optimal trajectory.

Title: Adaptive Deep Learning for High Dimensional Hamilton-Jacobi-Bellman Equations Authors: T. Nakamura-Zimmerer, Q. Gong, and W. Kang tenakamu@ucsc.edu, qigong@soe.ucsc.edu, wkang@gmail.com Abstract: Computing optimal feedback controls for nonlinear systems generally requires solving Hamilton-Jacobi-Bellman (HJB) equations, which, in high dimensions, is a wellknown challenging problem. In this talk, we present a data-driven method to approximate semi-global solutions to HJB equations for general high dimensional nonlinear systems and compute optimal feedback controls in real-time. To accomplish this, we model solutions to HJB equations with neural networks (NNs) trained on data generated without any state space discretization. Training is made more effective and efficient by leveraging the known physics of the problem and using the partially-trained NN to aid in adaptive data generation. We demonstrate the effectiveness of our method by learning solutions to HJB equations corresponding to the stabilization of a six dimensional nonlinear rigid body, and nonlinear systems of dimension up to 30 arising from the control of Burgers'-type partial differential equations. The trained NNs are then applied to implement real-time optimal feedback control of these systems.

Speaker: Dante Kalise, Imperial College d.kalise-balza@imperial.ac.uk Title: Taming the curse of dimensionality in Hamilton-Jacobi-Bellman equations by polynomial approximation and tensor calculus techniques. Abstract:

Title: Exploring the Sparsity of Error Covariance in Data Assimilation

Wei Kang, Department of Applied Mathematics, Naval Postgraduate School, Monterey, CA, USA

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Abstract: High resolution models of dynamical systems, like those used for weather prediction, may have tens of millions of state variables. The extremely high dimension results in an intractable error covariance because of the required high computational cost and I/O loads, as well as a large memory size needed in the process of the matrix. In data assimilation, the sparsity of error covariance is a fundamental property for the success of some widely used Kalman filter type methods. In this project, we explore the sparsity pattern and its computational algorithms for the error covariance of Kalman filters. We prove a relationship between the error covariance and a matrix upper bound in the form of controllability Gramian. Computational algorithms are developed so that elements in the matrix upper bound can be computed individually without the need of computing the entire matrix or a submatrix. As a result, the computational algorithms are component-based, i.e., the quantitative characteristics of sparsity pattern are computed over any given component in the matrix, such as a submatrix, a block, a row/column, etc. This property makes it possible to approximate error covariance on relatively small focused regions with high correlation without developing the entire covariance matrix, which is computationally intractable.

Title: Developments in Computational Approaches for Model Predictive Control Speaker: Ilya Kolmanovsky, Department of Aerospace Engineering, The University of Michigan, <u>ilya@umich.edu</u>

Speaker: Arthur J. Krener, NPS and UCD, <u>ajkrener@ucdavis.edu</u> Title: Nonlinear Systems Toolbox Abstract:

The Nonlinear Systems Toolbox is a package of MATLAB .m and .mex files for the analysis and synthesis of nonlinear control systems described by polynomials. Some of these .m files are implemented by .mex files so they run fast.

The basic data type is a vector field which is polynomial of arbitrary degree in a vector which is composed of an arbitrary number of subvectors. Of course, memory and speed limitations will implicitly restrict the degrees of the vector fields and dimensions of vectors. NST also works with polynomial matrix fields, polynomial higher order tensor fields and complex fields.

Speaker: William McEneaney, UCSD, <u>wmceneaney@eng.ucsd.edu</u> Title: Conversion of certain stochastic control problems into deterministic control problems

Abstract:

We consider nonlinear optimal stochastic control problems where the finitedimensional dynamics take the form of stochastic differential equations. These problems are typically converted into Hamilton-Jacobi partial differential equation (HJ PDE) problems. In the case of deterministic optimal control problems, the HJ PDEs are first-order equations, while in the stochastic case, these are second-order HJ PDEs.

In the case of first-order HJ PDEs, recent efforts have generated several numerical approaches that greatly attenuate the curse of dimensionality, which has severely hampered solution of nonlinear control problems for well over a half-century. The results have been quite promising in the case of first-order HJ PDEs, but less so for second-order HJ PDEs. Here, we demonstrate that for certain classes of problems, one may convert the second-order HJ PDE problems associated to control problems driven by stochastic differential equations into first-order HJ PDE problems and their associated deterministic control problems, thereby potentially allowing for much more substantial progress on nonlinear stochastic control problems.

Title: Approximate Model Predictive Control with Guaranteed Robustness and Feasibility

Speaker: Ali Mesbah, University of California Berkeley, mesbah@berkeley.edu Abstract: There is a growing interest in model predictive control (MPC) approaches with provably safe and robust performance that have low online computational and memory requirements, so that they can be deployed at high measurement sampling rates (i.e., possibly on sub microsecond sampling timescales) under resource (i.e., power and memory) constraints. Such MPC approaches would create unprecedented opportunities for real-time control of a wide range of emerging safety-critical and high-precision engineering applications, for example, in advanced manufacturing, biomedical systems, robotics, and unmanned vehicles, using low-power and memory embedded systems. In this talk, we will discuss recent advances in the area of explicit MPC that relies on precomputing explicit control laws to alleviate the need for online optimization. In particular, we will focus on machine learning-based approximate approaches to explicit MPC that can create new opportunities for fast and embedded MPC applications for constrained systems with a high state dimension and a large number of constraints. We will give an overview of the theoretical and computational challenges of approximate explicit MPC.

Title: Estimation of distributed parameter systems Speaker: Kirsten Morris, U. Waterloo, kmorris@uwaterloo.ca

Speaker: Stan Osher, UCLA, sjo@math.ucla.edu

Title: Fast numerical methods for mean field games and optimal transport and the link with Hamilton-Jacobi equations

Abstract:

We establish connections between optimal transport (OT) and mean field games (MFG). We develop unnormalized versions of both, allowing us to simply model and solve these by adding a source term into the continuity equation. We develop variational methods to solve these efficiently, including non-smooth energies. Hamilton-Jacobi equations arise naturally.

This is joint work with many people

Title: Optimization based state estimation; robustness analysis by Q functions Authors: James B. Rawlings and Douglas A. Allan Abstract.

State estimation can be posed as an optimal control/tracking problem. From this perspective, the stability and robustness properties of the estimator should be derivable from the properties of the optimization problem, as is commonly done in the analysis of Model Predictive Control. To achieve this goal in state estimation, we introduce a Lyapunov-like function, termed a Q function, and show that for general nonlinear systems satisfying a nonlinear detectability assumption and a nonlinear, incremental stabilizability assumption, the optimal full information state estimate is robustly asymptotically stable in the presence of bounded process and measurement disturbances. We also show that the state estimate converges to zero for asymptotically convergent disturbances.

These general theoretical results are illustrated by application to some numerical examples using the freely available software CasADi/MPCTools for solving the optimal control problems. Implications of these full information results for moving horizon estimation are also discussed.

Title Computational Optimal Control, *Theory and Tools Beyond Nonlinear Programming* Speaker I. M. Ross, Distinguished Professor

Naval Postgraduate School, Monterey, CA 93943, imross@nps.edu Abstract: Is an optimal control problem merely a large-scale nonlinear programming problem? Is the passage (i.e., map) from optimal control to nonlinear programming bijective? A version of these questions was framed in the 1960s after the announcement of the Maximum Principle; however, it is worth a second look 50+ years later. We conjecture a direct bijective passage from optimal control to nonlinear programming does not exist and frame it as a "theorem of impossibility." This provocative conjecture explains why nonlinear programming tools fail to solve certain practical optimal control problems.

We present optimal control programming – a fundamentally different algorithmic paradigm that avoids, in principle, the shortcomings of direct nonlinear programming methods. To address the "space of all optimal control problems," we introduce a tangent variable that transforms all nonlinear differential equations to a simple linear one. The cost of this transformation is the absence of a linearly transformed cotangent vector. By paying the price of a dual transformation, it is possible to maintain linearity in the cotangent subspace. In principle, one can then use primal-dual programming to solve the resulting problem. In this presentation, we show that there exists at least one primal discretization method that preserves linearity in the cotangent subspace. We show the existence of this discretization by direct construction using a Birkhoff interpolant. Significant computational efficiencies are possible when Birkhoff interpolants are constructed over the space of orthogonal polynomials. The resulting method exhibits a scale-invariant condition number leading to the possibility of a candidate optimal control programming method that is fast and accurate.

Title: Consistent Approximations in Optimization

Johannes O. Royset, Operations Research Department, Naval Postgraduate School, joroyset@nps.edu

Abstract:

Approximation is central to many optimization problems and the supporting theory provides insight as well as foundation for algorithms. In this talk, we lay out a broad framework for quantifying approximations by viewing finite- and infinite-dimensional constrained minimization problems as instances of extended real-valued lower semicontinuous functions defined on a general metric space. Since the Attouch-Wets distance between such functions quantifies epi-convergence, we are able to obtain estimates of optimal solutions and optimal values through bounds of that distance. In particular, we show that near-optimal and near-feasible solutions are effectively Lipschitz continuous with modulus one in this distance. Applications in optimal control and nonparametric statistics illustrate the theoretical results.

Title: Model Predictive Control of Hybrid Dynamical Systems Speaker: Ricardo Sanfelice Professor, Electrical and Computer Engineering

University of California, Santa Cruz,<u>https://hybrid.soe.ucsc.edu</u> Abstract:

Hybrid systems model the behavior of dynamical systems in which the states can evolve continuously and, at isolate time instances, exhibit instantaneous jumps. Such systems arise when control algorithms that involve digital devices are applied to continuous-time systems, or when the intrinsic dynamics of the system itself has such hybrid behavior, for example, in mechanical systems with impacts, switching electrical circuits, spiking neurons, atc. Hybrid control may be used for improved performance and robustness properties compared to conventional control, and hybrid dynamics may be unavoidable due to the interplay between digital and analog components in a cyberphysical system. In this talk, we will introduce analysis and design tools for model predictive control (MPC) schemes for hybrid systems. We will present recently developed results on asymptotically stabilizing MPC for hybrid systems based on control Lyapunov functions. After a short overview of the state of the art on hybrid MPC, and a brief introduction to a powerful hybrid systems framework, we will present key concepts and analysis tools. After that, we will lay out the theoretical foundations of a general MPC framework for hybrid systems, with guaranteed stability and feasibility. In particular, we will characterize invariance properties of the feasible set and the terminal constraint sets, continuity of the value function, and use these results to establish asymptotic stability of the hybrid closed-loop system. To conclude, we will illustrate the framework in several applications and summarize some of the open problems, in particular, those related to computational issues.

Title: Sensitivity Analysis, Uncertainty Quantification, and Control Design for Smart Material Systems

Speaker: Ralph C. Smith, North Carolina State University, rsmith@unity.ncsu.edu Abstract: This presentation will focus on concepts pertaining to sensitivity analysis (SA), uncertainty quantification (UQ), and control design for smart materials and adaptive structures. Pertinent issues will first be illustrated in the context of applications utilizing piezoelectric and shape memory alloy actuators, finite-deformation viscoelastic models, a fractional-order model for viscoelastic materials, and quantum-informed continuum models. The use of data, to improve the predictive accuracy of models, is central to uncertainty quantification so we will next provide an overview of how Bayesian techniques can be used to construct distributions for model inputs. The discussion will subsequently focus on computational techniques to propagate these distributions through complex models to construct prediction intervals for statistical quantities of interest such as expected displacements in macro-fiber composites and strains in SMA tendons. The use of sensitivity and active subspace analysis to isolate critical model inputs and reduce model complexity is synergistic with uncertainty quantification and will be discussed next. The presentation will conclude with discussion detailing how uncertainty quantification can be used to improve robust control designs for smart material systems.

Abstract: Arguably, one of the hardest challenges faced now by the control community stems from the exponential explosion in the availability of data, fueled by recent advances in sensing and actuation capabilities. Simply stated, classical techniques are ill equipped to handle very large volumes of (heterogeneous) data, due to poor scaling properties, and to impose the structural constraints required to implement ubiquitous sensing and control. For example, the powerful Linear Matrix Inequality framework developed in the past 20 years and associated semidefinite program based methods have proven very successful in providing global solutions to many control and identification problems. However, typically these methods break down when considering problems involving just a few hundred data points. Similarly, polynomial and semi-algebraic optimization techniques offer the promise of being able to move beyond quadratic Lyapunov functions in non linear control, but computational complexity and scaling issues limit their application to systems with relatively few states.

The goal of this talk is to explore how scalable, computationally tractable algorithms can be obtained by exploiting an underlying sparse structure, induced by the dynamics of the system. By appealing to a deep connection to semi-algebraic optimization and rank-preserving matrix completions, we will show that, in the context of systems theory, the limiting factor is given by the ``memory" of the system rather than the size of the data itself, and discuss the implications of this fact. These concepts will be illustrated by examining examples of "easy" and "hard" problems, including identification of switched systems and data driven control of polynomial systems. We will conclude the talk by briefly examining recent developments that hold the promise of being able to efficiently solve the polynomial optimization problems arising in these contexts by solving a sequence of sparse second order cone programs.

Speaker: Daniele Venturi, UCSC, venturi@ucsc.edu

Title : Hierarchical tensor methods for high-dimensional nonlinear PDEs Abstract :

In this talk I will present a new method to compute the numerical solution of highdimensional nonlinear PDEs on low-rank tensor manifolds. The key idea relies on a hierarchical decomposition of the solution space in terms of a sequence of nested subspaces of smaller dimension. This process, which can be conveniently be visualized in terms of binary trees, yields series expansions that include classical Tensor-Train and Hierarchical Tucker representations.By enforcing dynamic orthogonality conditions at each level of the binary tree representing the solution tensor, we obtain coupled evolution equations for the tensor modes spanning each subspace. This allows us to compute the numerical solution of high-dimensional time-dependent PDEs on tensor manifolds with constant rank, with no need for computationally expensive rank reduction methods. I will also discuss new algorithms for dynamic addition and removal of modes and present numerical examples involving high-dimensional hyperbolic and parabolic PDEs The dynamic programming formulation of optimal control problems leads to Hamilton-Jacobi PDEs, which are often considered prohibitively expensive due to a "curse of dimensionality". But there are many challenges remaining even in low-dimensional state spaces, where relatively fast and accurate algorithms are well-developed for canonical problems. These challenges include not only the

efficiency/accuracy/scalability trade-offs, but also many fundamental model features useful for practitioners: multivalued solutions, multiple criteria of optimality,

uncertainties, different models of stochastic perturbations, and different notions of "robustness". In this talk, I will illustrate these issues on several simple examples with isotropic running cost(s) and dynamics.

Speaker: Linda Wang, U. Texas, Arlington, shuolinda.wang@uta.edu

Title: Conjugate cases in optimal ensemble controls

Abstract: Controlling a population system consisting of a large number of structural identical dynamic units is an essential step that enables many cutting-edge applications in science and engineering, such as optimal pulse design for exciting quantum systems between the desired states and neural stimulation for alleviating the symptoms of neuronal disorders. Ensemble control, which guides this large population of nonlinear dynamic systems to achieve a desired control task, needs to be robust and accurate. Existing computational methods for solving optimal ensemble control problems, such as shooting method and pseudo-spectral method, may encounter low efficiency, slow convergence, and instable issues. In this work, we discuss the conjugate cases in finding optimal ensemble controls, which violate the second-order optimal conditions. We use a couple of applications to illustrate the idea of ensemble controls, one of which presents the conjugate case.

Speaker: Hasnaa Zidani, hasnaa.zidani@ensta-paris.fr

Title: "The relationship between Pontryagin's principle and

Hamilton-Jacobi approach. Application to some aerospace problems." Abstract: "

In optimal control theory, it is well known that the costate variable of the Pontryagin principle can be interpreted in term of gradient of the value function, evaluated along the optimal state trajectory. This relationship is well established when the problem is without state constraints. In presence of state constraints, even very small perturbations of the optimal control can give trajectories that violate the constraints and the value function is in general discontinuous. In this talk, we will discuss a general sensitivity result for some control problems with state constraints. Then we will show how to combine this sensitivity relation with the shooting method on some aerospace problems.

Poster Presentations

TITLE: CONTROL-THEORETIC MODELS OF ENVIRONMENTAL CRIME

PRESENTER; ELLIOT CARTEE, Cornell U., evc34@cornell.edu

Title: Power and Limitations of Algebraic Proofs of Stability Based on Semidefinite Programming

Presenter: Bachir El Knadir, Princeton U., bkhadir@princeton.edu Abstract: We study the power and limitations of sum of squares optimization and semialgebraic Lyapunov functions for proving asymptotic stability of polynomial dynamical systems. We give the first example of a globally asymptotically stable polynomial vector field with rational coefficients that does not admit a polynomial (or even analytic) Lyapunov function in any neighborhood of the origin. We show, however, that if the polynomial vector field is homogeneous, then its asymptotic stability is equivalent to existence of a rational Lyapunov function whose inequalities have sum of squares proofs. This statement generalizes the classical result in control on the equivalence between asymptotic stability of linear systems and existence of a quadratic Lyapunov function satisfying a certain linear matrix inequality. (Based on joint work with Amir Ali Ahmadi.)

Presenter: Wuchen Li, UCLA, wcli@math.ucla.edu

Title: Mean field control, game and beyond.

Abstract: Optimal control in probability distributions plays essential roles in physics and data science. By varying differential initial and terminal control conditions, several famous examples are presented, including optimal transport, mean field games and Schrodinger bridge problems. In this poster, we present the fast algorithms therein, with various applications in robotics path planning and learning algorithms.

This is based on joint works with Wuchen Li, Stanley Osher and team members in ASFOR MURI.

Presenter: Alex Tong Lin, UCLA, atlin@math.ucla.edu

Title: Splitting for Hamilton-Jacobi Equations Arising from Optimal Control and Differential Games

Abstract: In this work, we apply a new splitting method based on the Primal Dual Hybrid Gradient algorithm (a.k.a. Chambolle-Pock) to nonlinear optimal control (OC) and differential games (DG) problems, based on using the direct collocation method, but with a Hamiltonian twist. This allow us to compute solutions at specified points directly, i.e. without the use of grids in space. And it also gives us the ability to create trajectories directly. Thus we are able to lift the curse of dimensionality a bit, and therefore compute solutions in much higher dimensions than before. And in our numerical experiments, we actually observe that our computations scale polynomially in time. Furthermore, this new algorithm is embarrassingly parallelizable.