

## Lab 5: SYSTEM STABILITY

### Section 1 – Background

The Routh-Hurwitz criterion is a method by which one can investigate the stability of a system. In this lab the stability of the torsional control plant will be investigated. Starting with the conditions for stability as outlined by the Routh-Hurwitz criterion, a range of gain values for the position feedback,  $K_p$ , and the velocity feedback,  $K_d$ , will be obtained. From these results, the stability of the actual plant will be explored.

### Section 2 – Procedure

#### 2.1 Determine System Stability (Prelab Work)

Figure 1 shows the block diagram of the torsion control plant used in the lab (neglect Dead Zone). Through selection of the gains  $K_p$  and  $K_d$ , the system can be made to produce any type of response—overdamped, underdamped, even an unstable response can be produced. Through the application of the Routh-Hurwitz criterion, a range of values for the position feedback,  $K_p$ , and the velocity feedback,  $K_d$ , will be established. See accompanying notes at the end of this procedure.

2.1.1 Starting with the block diagram of the torsional control plant, reduce it to an equivalent closed-loop transfer function. Apply the Routh-Hurwitz criterion to the denominator of this function to determine the conditions for stability.

2.1.2 Make a graph similar to the one shown in Figure 2. Indicate where the system is stable and where it is unstable

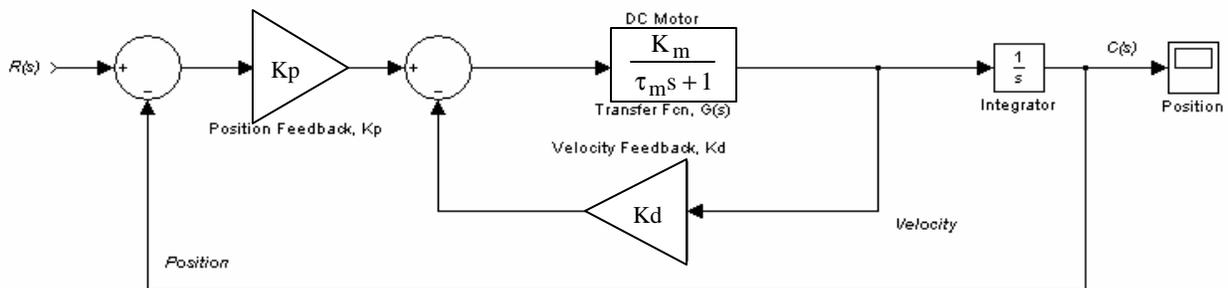


Figure 1: Block Diagram for Torsional Control Plant

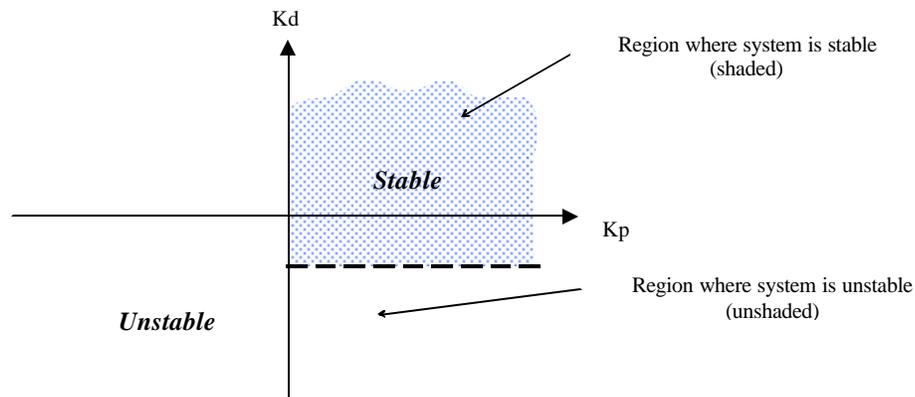


Figure 2: Graph of  $K_d$  versus  $K_p$  Showing Regions of Stability

## 2.2 Demonstrate Stable System Response with the Real System

From the results of the Routh-Hurwitz criterion, the torsional control system is found to be stable for the conditions where  $K_p > 0$  and  $K_d > -1/K_m$ . Using these conditions, demonstrate that the system is stable when  $K_p$  and  $K_d$  are chosen appropriately.

2.2.1 Set up the Control Plant. The torsional system will be set up with two (2 each) weights loaded on the bottom disk only. The weights are secured 180 degrees apart from each other such that the outside edge of each weight is tangent to the 9-cm radius line (last line on the disk).

2.2.2 Start the ECP Executive software program.

2.2.3 Energize the control system by pushing the "ON" button on the ECP controller.

### WARNING

*The system is now energized and will rotate at potentially high speeds when a control voltage is applied to the motor of the torsional system. At any point the motion of the system can be stopped by pressing the OFF (red) button on the ECP controller box.*

2.2.4 Setup the ECP Program as done in previous labs with the following variations:

- a. For Control Algorithm set
  - $K_p = 0.1$  (see CAUTION below)
  - $K_d = 0.002$  (see CAUTION below)

### CAUTION

*While the Routh-Hurwitz analysis indicates that the system is stable for  $K_p > 0$  and  $K_d > -1/K_m$ , the physical system has limits (i.e., motor over-speed, excessive shaft deflection, or motor drive current) which can not be exceeded. Thus, it is recommended that you use gain values known to produce stable results for this part of the procedure.*

- b. In the Trajectory window, select Closed Loop Impulse, then set
  - Amplitude (degrees)= 45
  - Pulse Width (msec) = 5000
  - Number of Reps = 1
  - Dwell Time (msec)= 0

2.2.5 Run the setup.

2.2.6 Plot Data: Select Plotting → Setup Plot. For the left axis choose Commanded Position and Encoder 1 Position. Include a copy of the plot in your report. Note the values for  $K_p$  and  $K_d$  on the plot.

## 2.3 Explore Conditions for Stability where Position Feedback is Positive ( $K_p > 0$ )

For this part of the procedure, set  $K_p > 0$ , but allow  $K_d$  to approach  $-1/K_m$ . In this manner, the borders of instability can be probed with caution.

2.3.1 Set Input Trajectory Amplitude. Set Amplitude (degrees) = 5. Only a small input is required to upset an unstable system.

2.3.2 Set System Gains.

- a. Set  $K_p = 0.1$
- b. Set  $K_d = 50\%$  of  $-1/K_m$
- c. Remember to Implement Algorithm

2.3.3 Run the program and plot the results. If the system does not produce the unstable response, gradually increase the magnitude (absolute value) of  $K_d$ , but keep the sign negative until you observe the unstable

system response. Include a plot in your report where the unstable response is observed. Note the corresponding values of  $K_d$  and  $K_p$  on the plot.

#### **2.4 Explore Conditions for Stability where Position Feedback is Negative ( $K_p < 0$ )**

From the results of the Routh-Hurwitz analysis, it was found that  $K_p$  must be positive ( $K_p > 0$ ) for a stable system, as well as that  $K_d > -1/K_m$ . For this final part of the procedure, we will set  $K_p$  to a negative value and demonstrate once again the unstable system response.

##### 2.4.1 Set System Gains.

- a. Set  $K_p = -0.001$
- b. Set  $K_d = 0.002$
- c. Remember to Implement Algorithm

2.4.2 Run the program and plot the results. If the system does not produce the unstable response, gradually increase the magnitude (absolute value) of the position feedback,  $K_p$ , but keep the sign negative until you observe the unstable system response. Include a plot in your report where the unstable response is observed. Note the corresponding values of  $K_d$  and  $K_p$  on the plot.

### **Section 3 -- Report**

Submit a report in the standard format for this class. In the Introduction discuss the relevant theory that was presented and explored during the lab. In the Results and Analysis show the formulation of the equivalent transfer function and your Routh-Hurwitz analysis. Be sure to include all of your plots.

Finally, discuss the main points of the lab in the Conclusion of the report. Include any significant findings and recommendations you may have.

### Notes for Lab 5

The closed loop transfer function of the torsion control system (Figure 1) is reduced to the form given in Equation 1.

$$T(s) = \frac{K_p K_m}{\tau_m s^2 + (1 + K_d K_m)s + K_p K_m} \quad \text{Equation 1}$$

The Routh-Hurwitz criterion for stability applied to the denominator of T(s) is shown in the table below.

$s^2$	$\tau_m$	$K_p K_m$
$s^1$	$1 + K_d K_m$	0
$s^0$	$-\frac{(\tau_m \cdot 0 - K_p K_m (1 + K_d K_m))}{1 + K_d K_m} \rightarrow K_p K_m$	

And for stability we get that

$$1 + K_d K_m > 0 \quad \rightarrow \quad K_d > -\frac{1}{K_m}$$

$$K_p K_m > 0 \quad \rightarrow \quad K_p > 0$$

Referring to the results of Lab 2 for the value of  $K_m$ , the conditions for stability are determined from the inequalities given derived above.