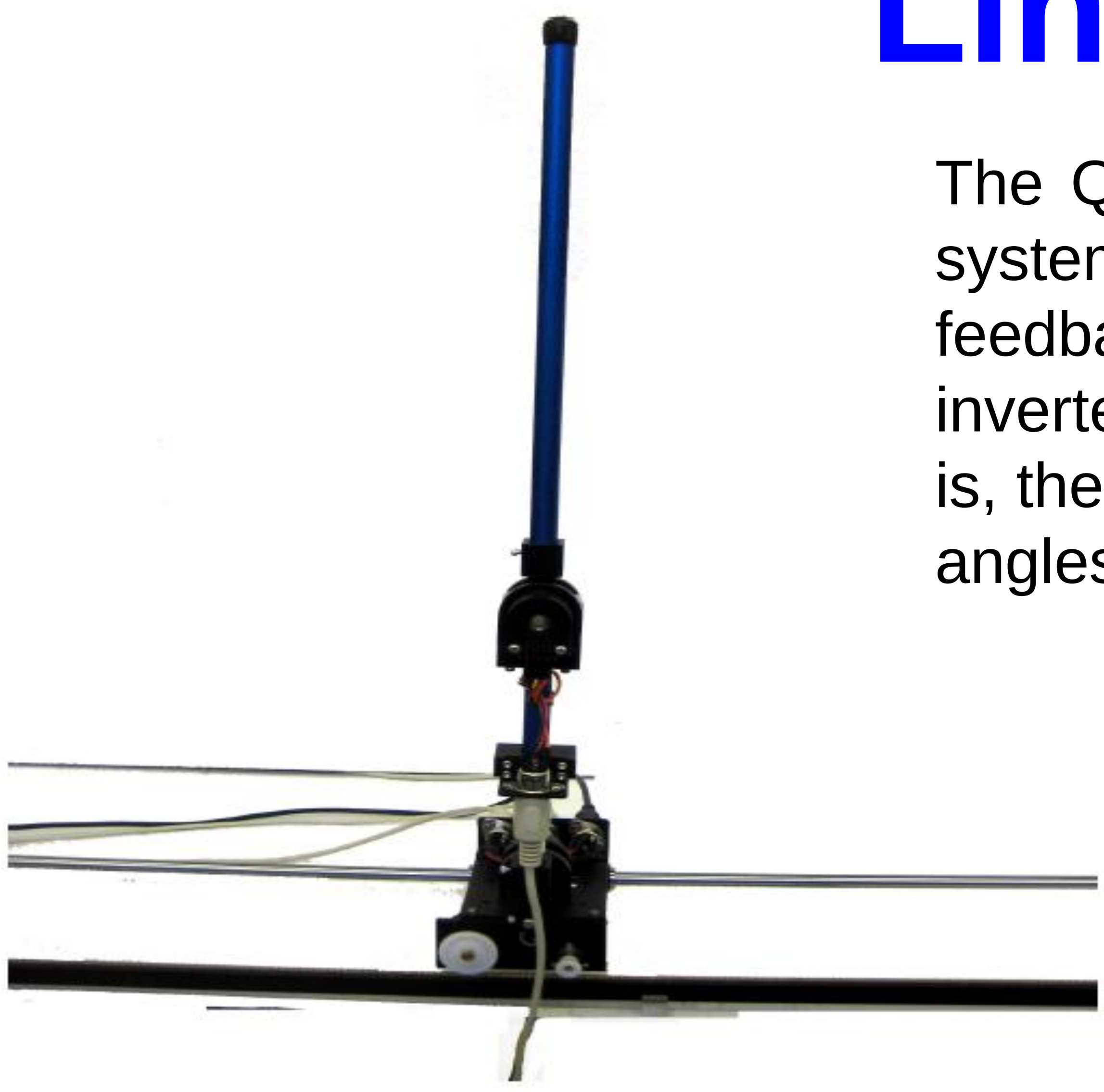
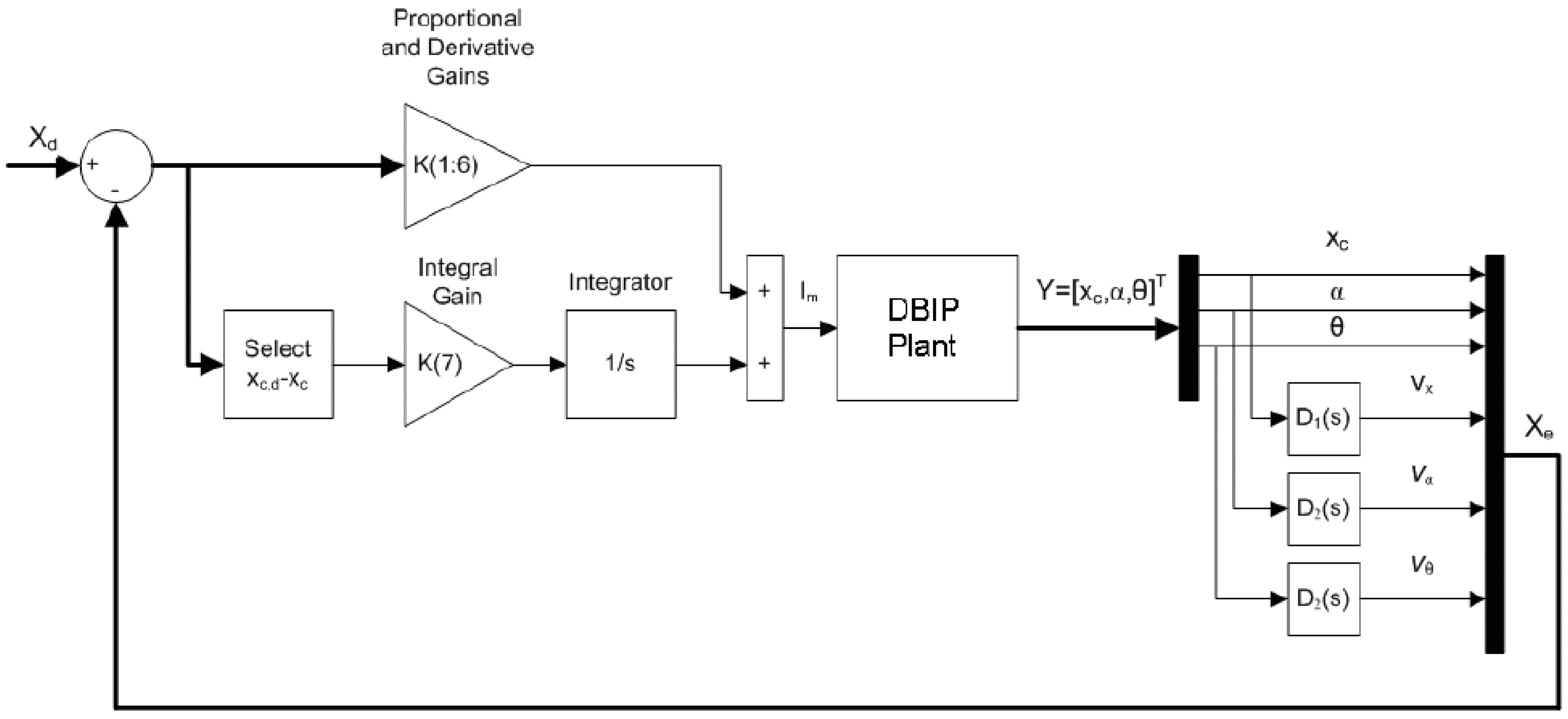
Linear Double Inverted Pendulum

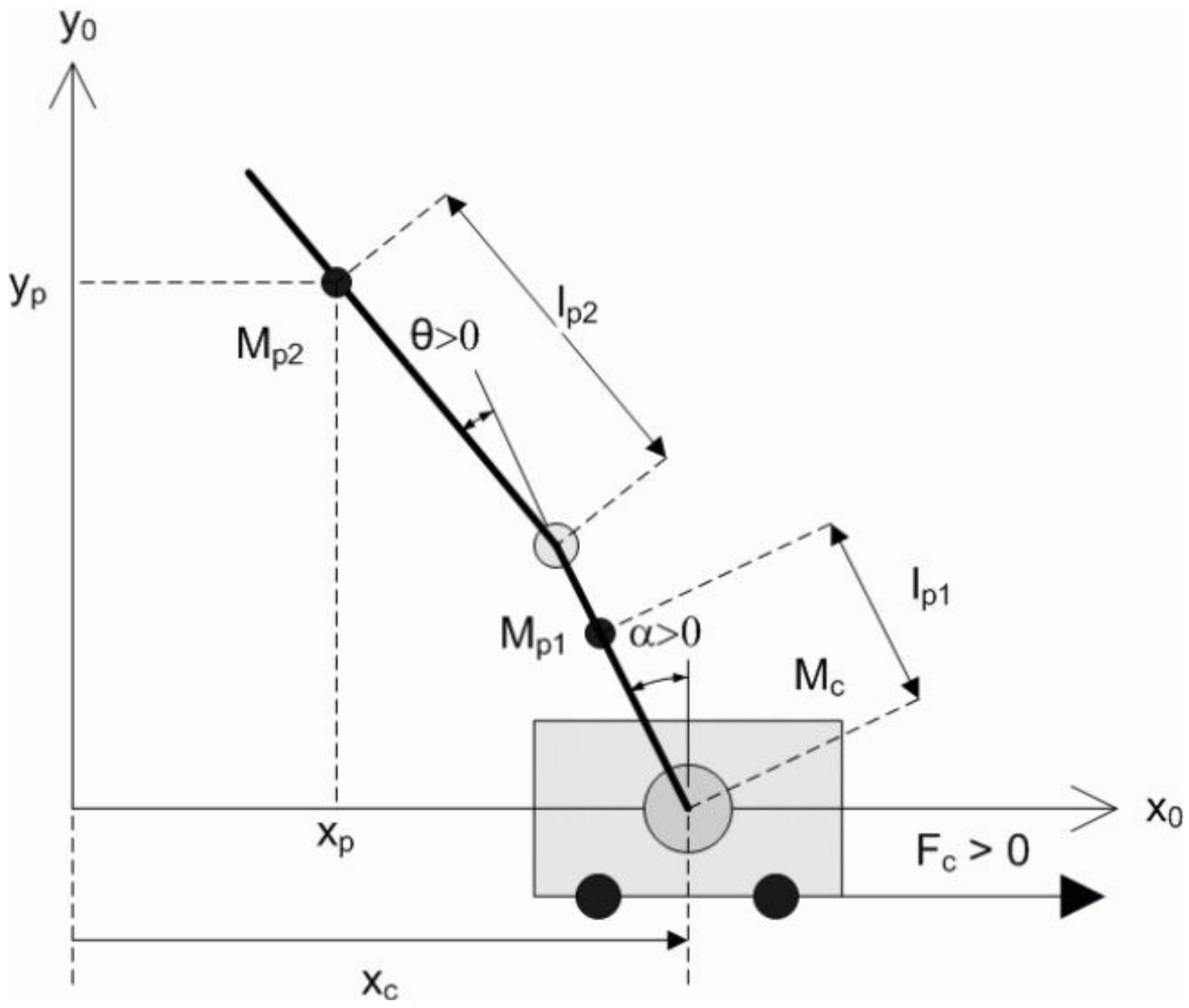


The Quanser Double Inverted Pendulum (DBIP) is an unstable, non-minimum phase system that is controllable through feedback. The objective is to design a state-feedback control system (LQR controller) that stabilizes the system keeping a double inverted pendulum balanced and tracking the IP02 cart to a commanded position. That is, the cart position x_c, tracks a desired setpoint position $x_{c,d}$, and both upright pendulum angles, α and θ, are stabilized about 0.

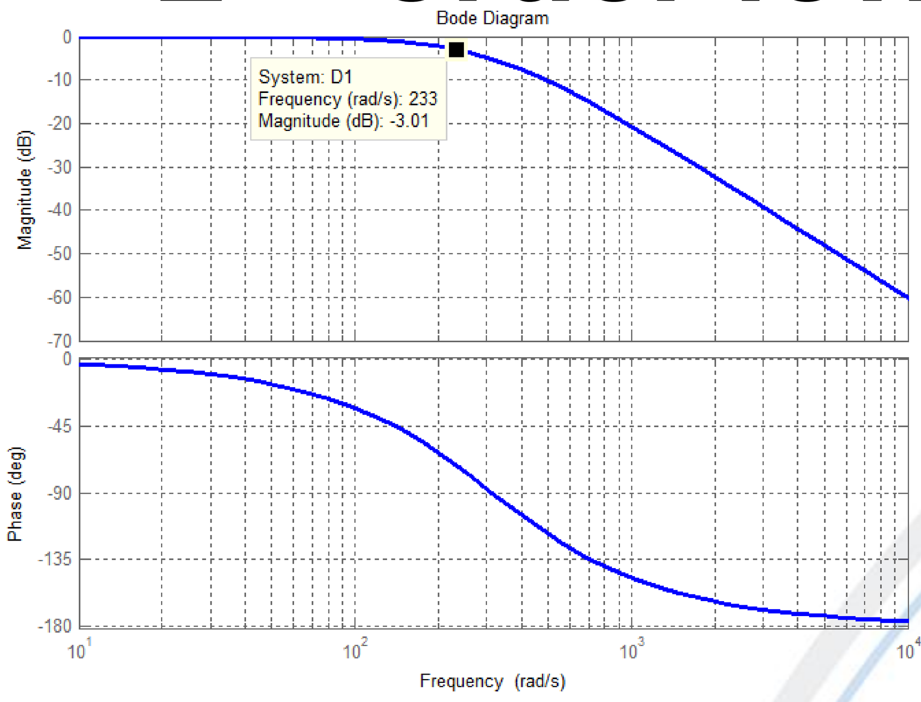
DBIP controller



DBIP free body diagram



2nd-order low-pass filter



$$\omega_{n_x} = 2\pi \cdot 50Hz$$

$$\omega_{n_{\alpha,\theta}} = 2\pi \cdot 10Hz$$

$$D_{1,2,3}(s) = \frac{\omega_n^2}{s^2 + 1.8\omega_n + \omega_n^2} s$$

Objective

$$\mathbf{x}(t) = \left[x_c - x_{c,d}, \alpha, \theta, \dot{x}_c, \dot{\alpha}, \dot{\theta}, \int (x_c - x_{c,d}) dt \right]^T \underset{t\to\infty}{\to} 0$$

Limitations

Cart position:	$x_c \le \pm 0.4m$
Lower pendulum angle:	$\alpha \le \pm 20°$
Upper pendulum angle:	$\theta \le \pm 20°$

LQR controller design

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu \qquad u = -K\mathbf{x} \qquad J = \int_0^\infty (\mathbf{x}^T Q \mathbf{x} + uRu) dt$$

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \qquad K = R^{-1} B^T P$$

Physical parameters are defined in: *setup_lab_ip01_2_dbip.m*

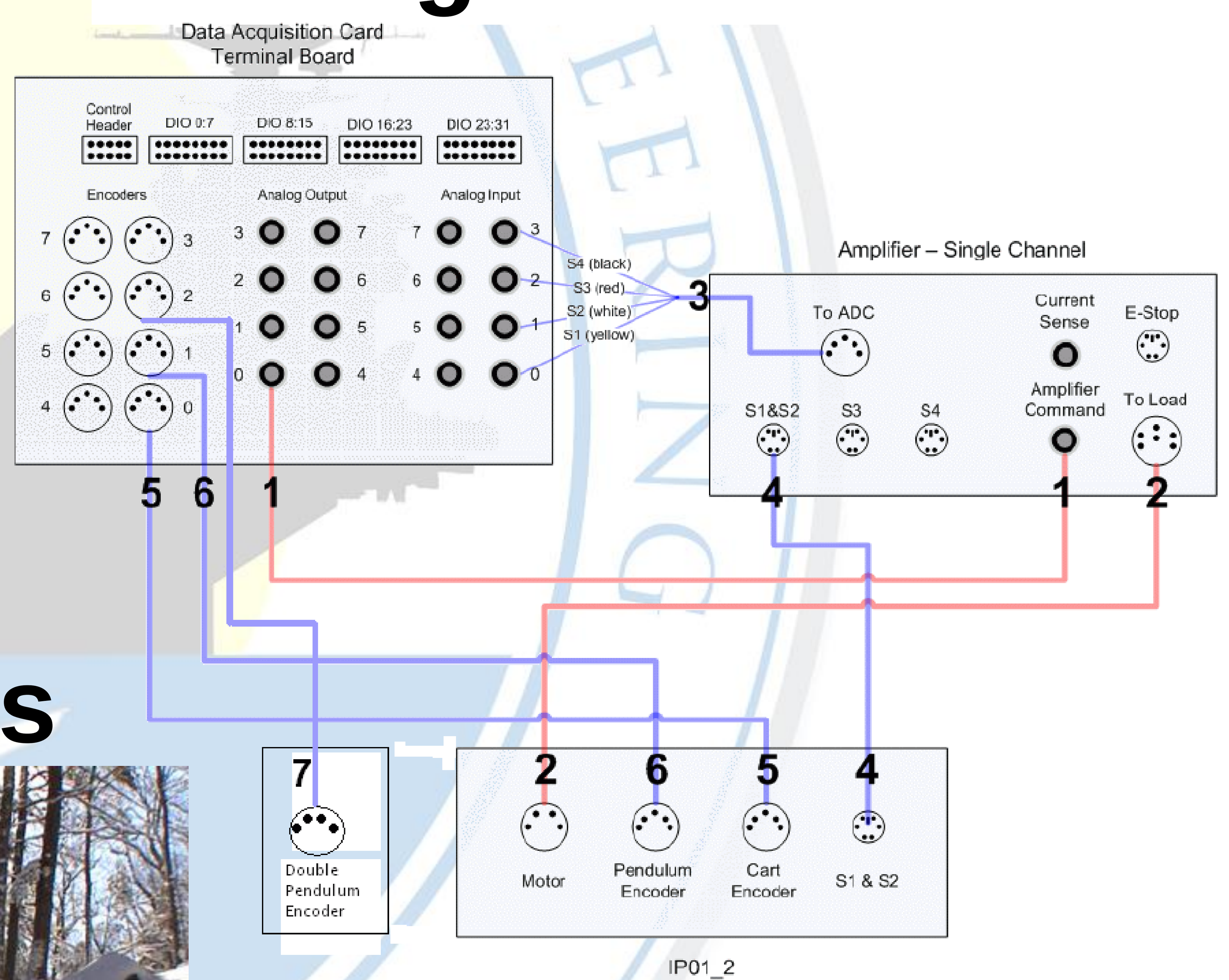
Matrices *A*, *B*, *C* and *D* are computed in: *DBIP_ABCD_eqns.m*

LQR controller gain computation: `K = lqr(A,B,Q,R);`

$$Q = diag([10, 50, 50, 0, 0, 0, 0.1]) \qquad R = 0.1$$

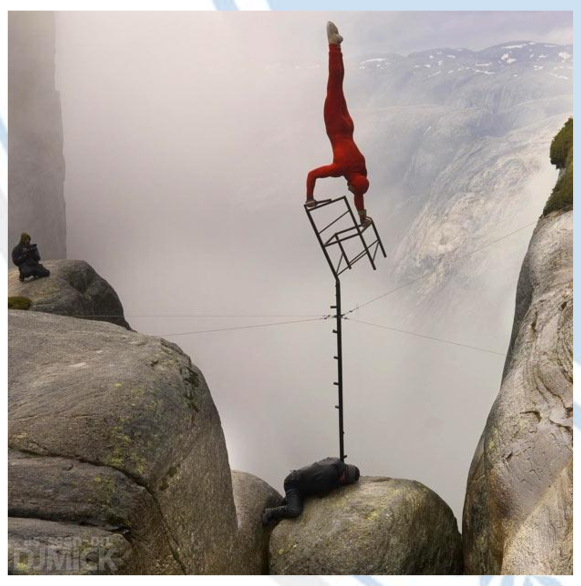
$$K = [11.1 \quad -51.3 \quad 136 \quad 7.7 \quad -12.6 \quad -12 \quad 1]$$

Wiring schematics



Applications

Eskil Ronningsbakken



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