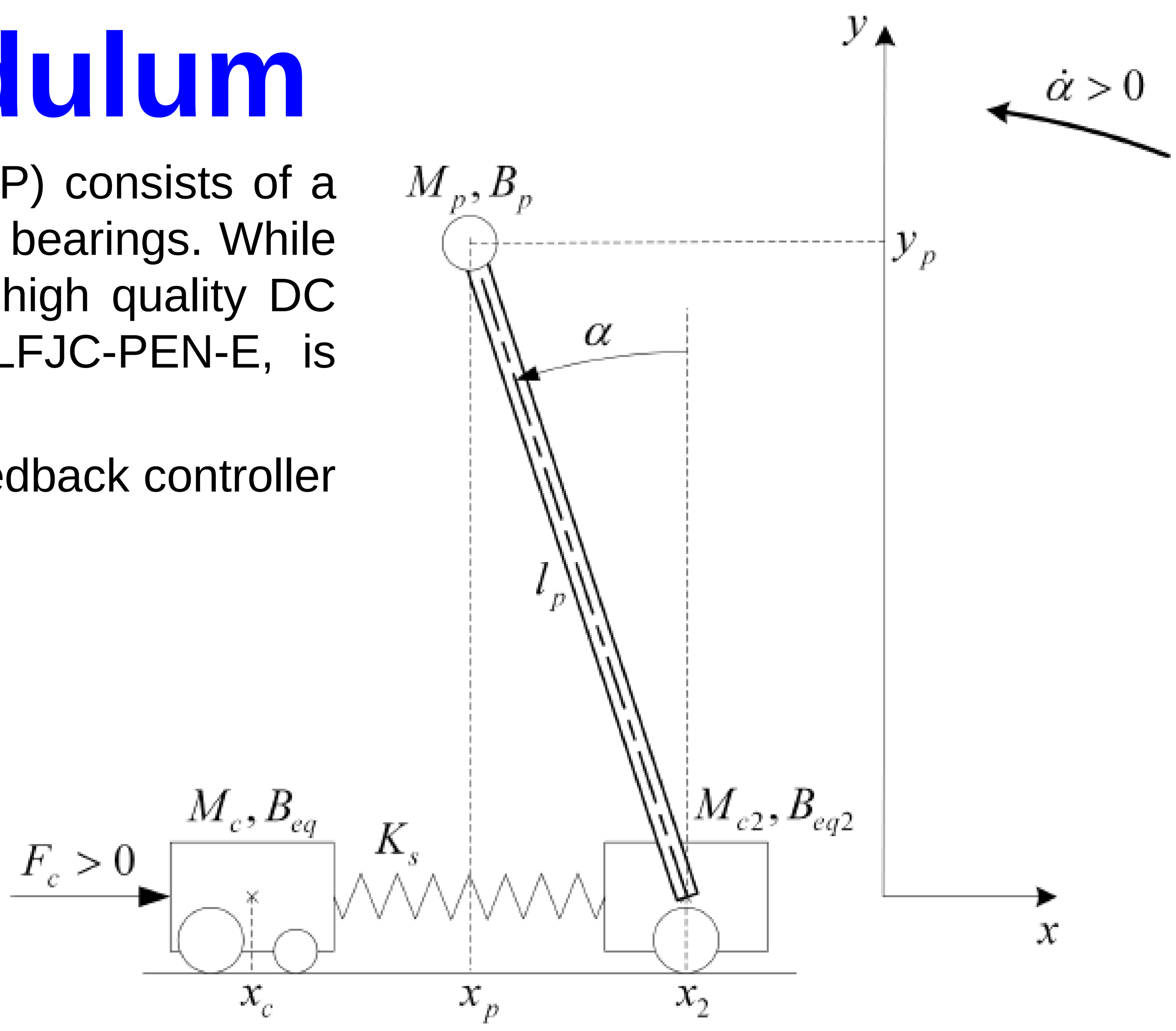
Linear Flexible Joint Cart plus Single Inverted Pendulum

Linear Flexible Joint Cart plus Single Inverted Pendulum (LFJC+SIP) consists of a system of two carts sliding along a stainless steel shaft using linear bearings. While one of the two carts is motorized and drives the system using a high quality DC motor equipped with a planetary gearbox, the second cart, a LFJC-PEN-E, is passive and coupled to the first one through a linear spring.

The objective is to design, simulate, and tune a LQR-based state-feedback controller satisfying the closed-loop system's desired design specifications.



Requirements

Maximum pendulum angle:	$\lvert\alpha\rvert \le 2.5°$
Maximum spring deflection:	$\lvert x_s\rvert \le 15mm$
Cart position:	$\lvert x_c\rvert \le 60mm$
Maximum voltage:	$\lvert V_m\rvert \le 4V$

State-space equations

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu$$

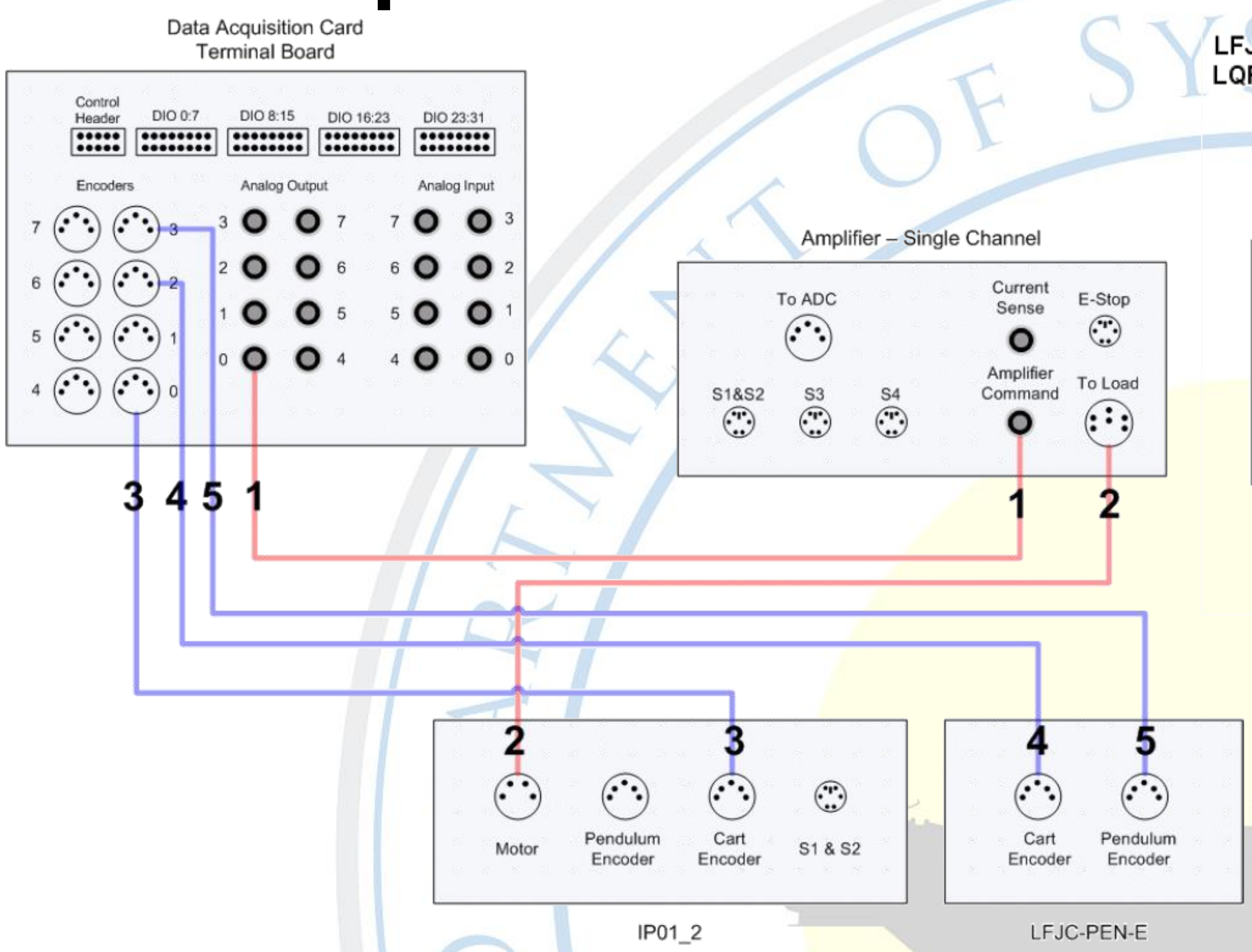
$$u = -K\mathbf{x}$$

$$\mathbf{x}(t) = \begin{bmatrix} x_c \\ x_s \\ \alpha \\ \dot{x}_c \\ \dot{x}_s \\ \dot{\alpha} \\ \int x_c dt \end{bmatrix} \underset{t\to\infty}{\to} 0$$

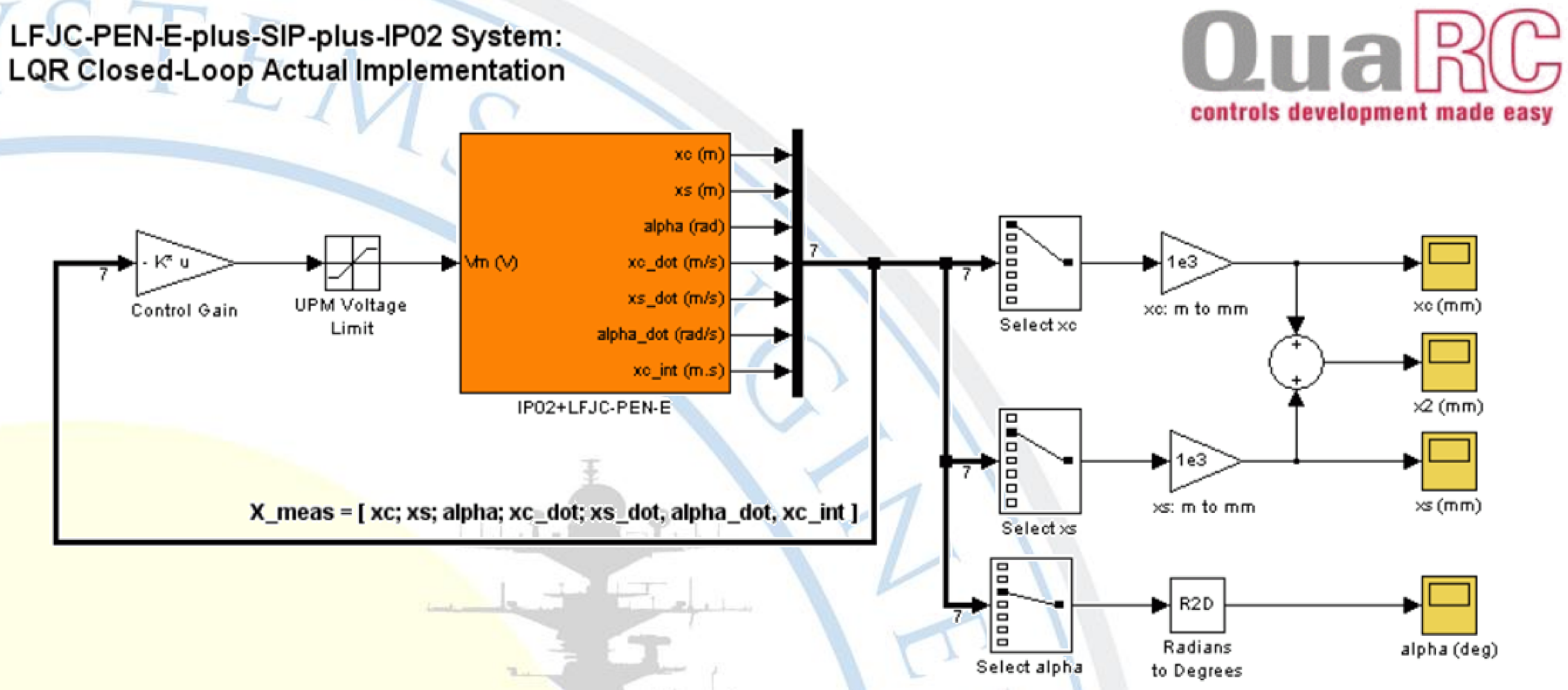
$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & K_s/M_c & 0 & -B_{eq}/M_c & 0 & 0 & 0 \\ 0 & -K_s/M_c - K_s/M_{c2} & M_p g/M_{c2} & -B_{eq}/M_c - B_{eq2}/M_{c2} & -B_{eq2}/M_{c2} & -B_p/(l_p M_{c2}) & 0 \\ 0 & -K_s/(l_p M_{c2}) & g(M_{c2}+M_p)/(l_p M_{c2}) & -B_{eq2}/(l_p M_{c2}) & -B_{eq2}/(l_p M_{c2}) & -B_p(M_{c2}+M_p)/(M_p l_p^2 M_{c2}) & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \left[0,0,0,\frac{1}{M_c},\frac{-1}{M_c},0,0\right]^T$$

Setup schematics



Simulink block diagram



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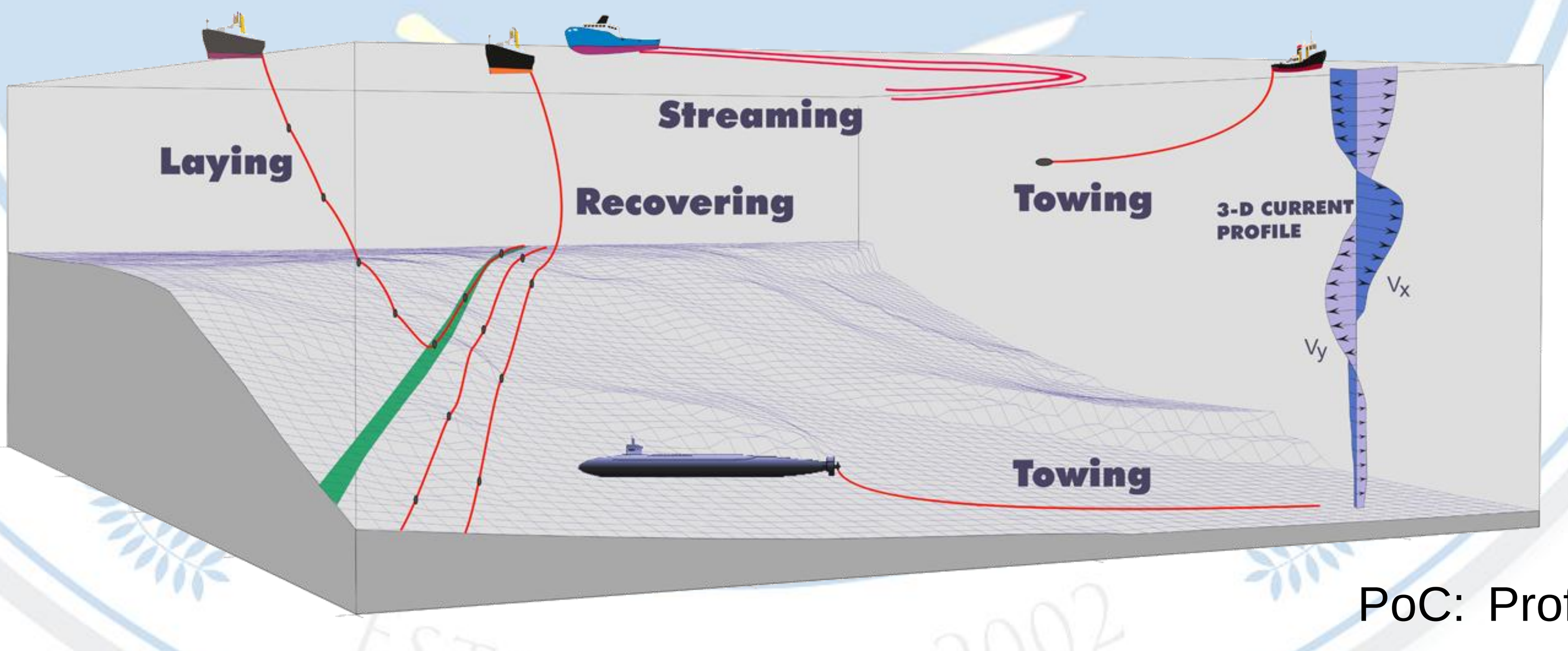
LQR controller

$$Q = diag([4000, 500, 3000, 0, 0, 0, 100]) \quad R = 0.25$$

$$K = [-139 \quad -409 \quad 304 \quad -96 \quad -117 \quad 50 \quad -20]$$

Military/Navy Applications



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