Rotary Inverted Pendulum

The Quanser Rotary Inverted Pendulum consists of a flat arm, or hub, with a pivot at one end and a metal shaft on the other end. The pivot-end can be mounted on top of the SRV02 load gear shaft and fastened with screws. The actual pendulum link is fastened onto the metal shaft and the shaft is instrumented with a sensor to measure its angle. The result is a horizontally rotating arm with a pendulum at the end.

The student is challenged to apply the Euler-Lagrange equations to find the system equations of motion, followed by their linearization and state-space representation. This experiment supports studying and demonstrating both dynamic and static instability concepts and practices in control engineering (controllability, companion matrix, pole placement, feedback control, swing-up control).

Equations of motion

$$(m_pL_r^2+\frac{1}{4}m_pL_p^2-\frac{1}{4}m_pL_p^2\cos(\alpha)^2+J_r)\ddot{\theta}-\frac{1}{2}m_pL_pL_r\cos(\alpha)\ddot{\alpha}+\frac{1}{2}m_pL_p^2\sin(\alpha)\cos(\alpha)\dot{\theta}\dot{\alpha}+\frac{1}{2}m_pL_pL_r\sin(\alpha)\dot{\alpha}^2=\tau-B_r\dot{\theta}$$

$$-\frac{1}{2}m_pL_pL_r\cos(\alpha)\ddot{\theta}-(J_p+\frac{1}{4}m_pL_p^2)\ddot{\alpha}-\frac{1}{4}m_pL_p^2\sin(\alpha)\cos(\alpha)\dot{\theta}^2-\frac{1}{2}m_pL_pg\sin(\alpha)=B_p\dot{\alpha}$$

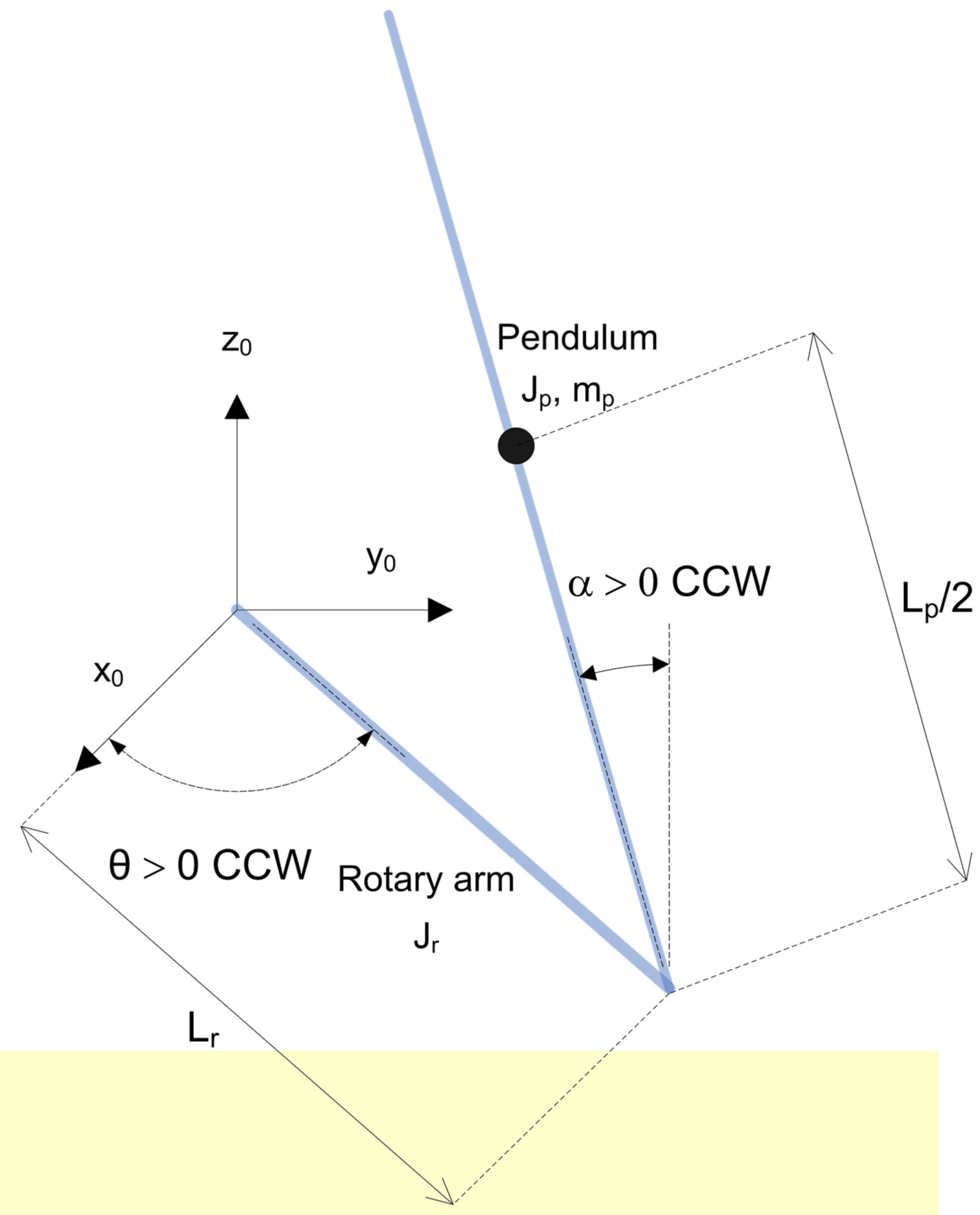
$$\tau=\frac{\eta_gK_g\eta_mk_t(V_m-K_gk_m\dot{\theta})}{R_m}$$

$$\dot{\mathbf{x}}=A\mathbf{x}+B\mathbf{u} \qquad \mathbf{x}=\begin{bmatrix}\theta\\ \alpha\\ \dot{\theta}\\ \dot{\alpha}\end{bmatrix} \qquad \mathbf{y}=\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

$$\mathbf{y}=C\mathbf{x}+D\mathbf{u}$$

$$A=\frac{1}{J_T}\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & \frac{1}{4}m_p^2L_p^2L_rg & -(J_p+\frac{1}{4}m_pL_p^2)B_r & -\frac{1}{2}m_pL_pL_rB_p\\ 0 & \frac{1}{2}m_pL_pg(J_r+m_pL_r^2) & \frac{1}{2}m_pL_pL_rB_r & (J_r+m_pL_r^2)B_p\end{bmatrix}$$

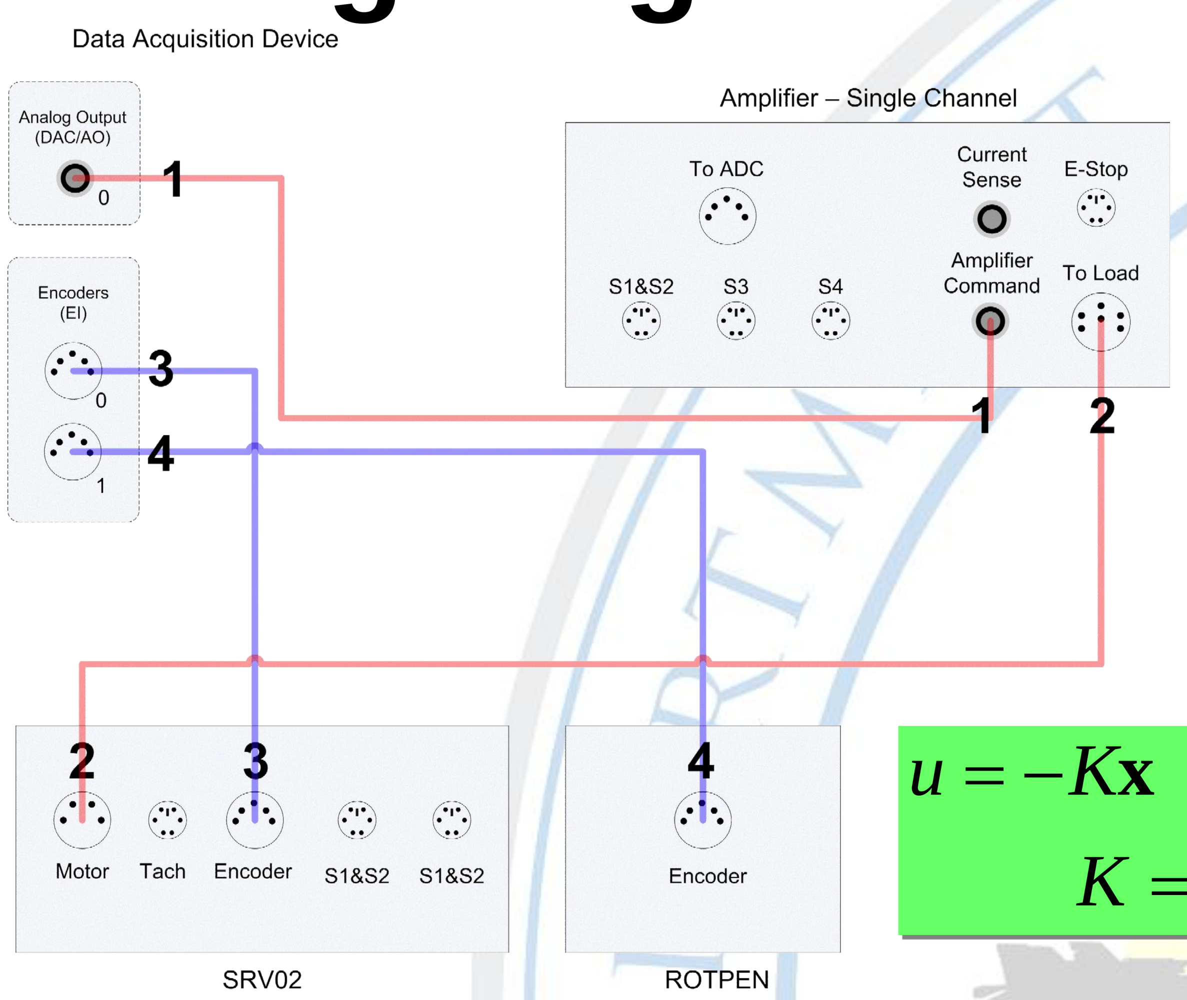
$$B=\frac{1}{J_pm_pL_r^2+J_rJ_p+\frac{1}{4}J_rm_pL_p^2}\begin{bmatrix}0\\ 0\\ J_p+\frac{1}{4}m_pL_p^2\\ \frac{1}{4}m_pL_pL_r\end{bmatrix} \qquad C=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix} \qquad D=\begin{bmatrix}0\\ 0\end{bmatrix}$$



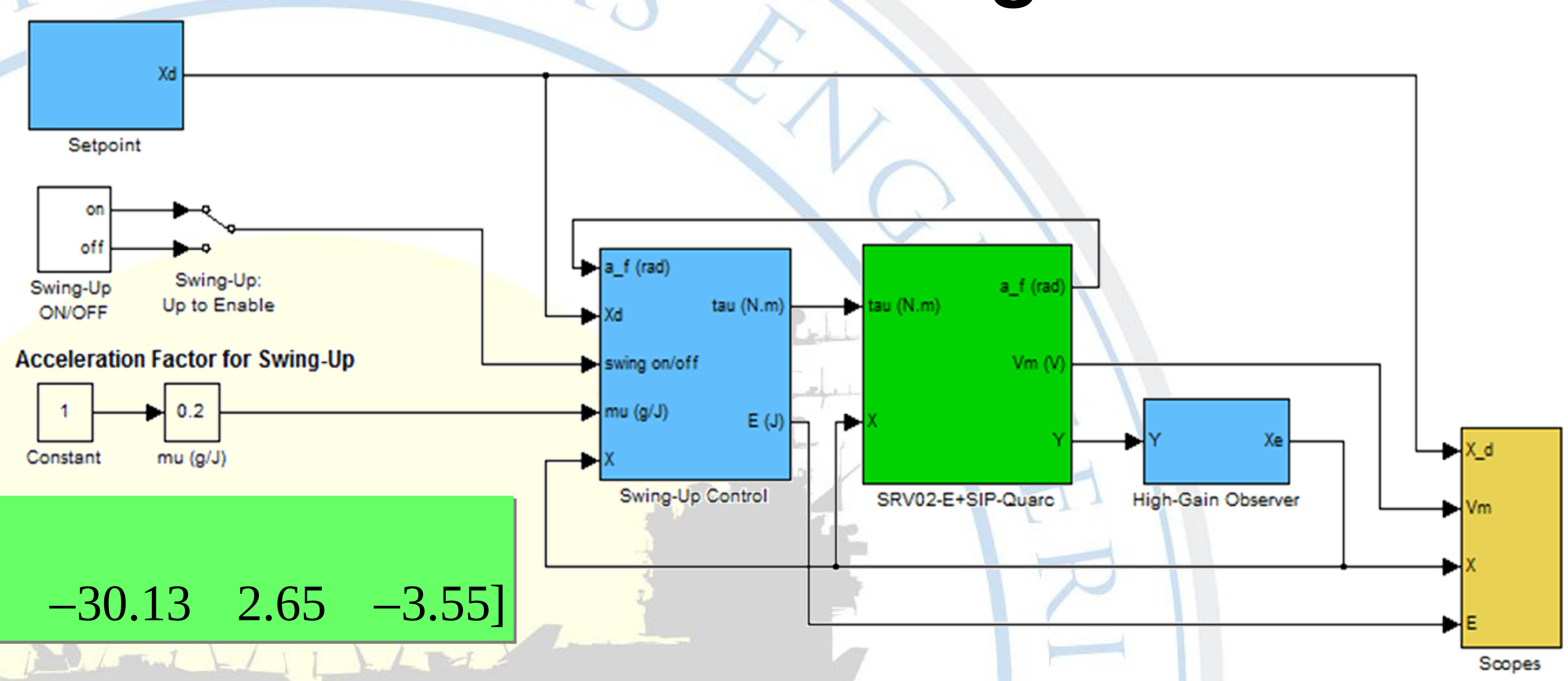
Specifications

Damping ratio:	$\zeta = 0.7$
Natural frequency:	$\omega_n = 4\ rad/s$
Maximum pendulum angle deflection:	$\lvert\alpha\rvert < 15°$
Maximum control effort / voltage:	$\lvert V_m\rvert < 10\ V$

Wiring diagram



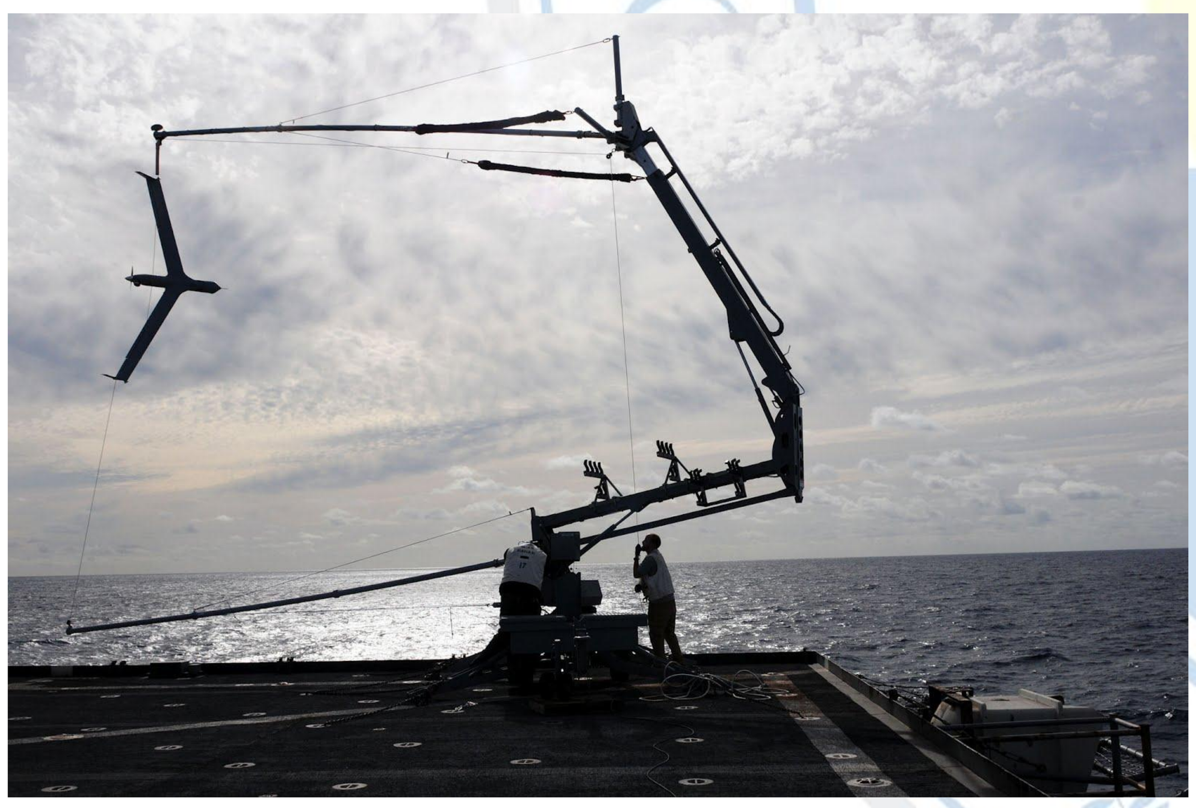
Simulink block diagram

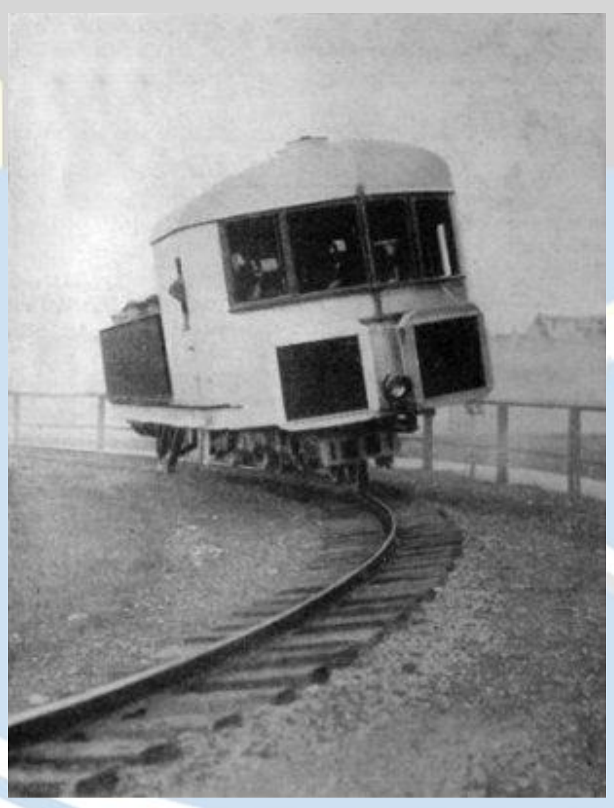


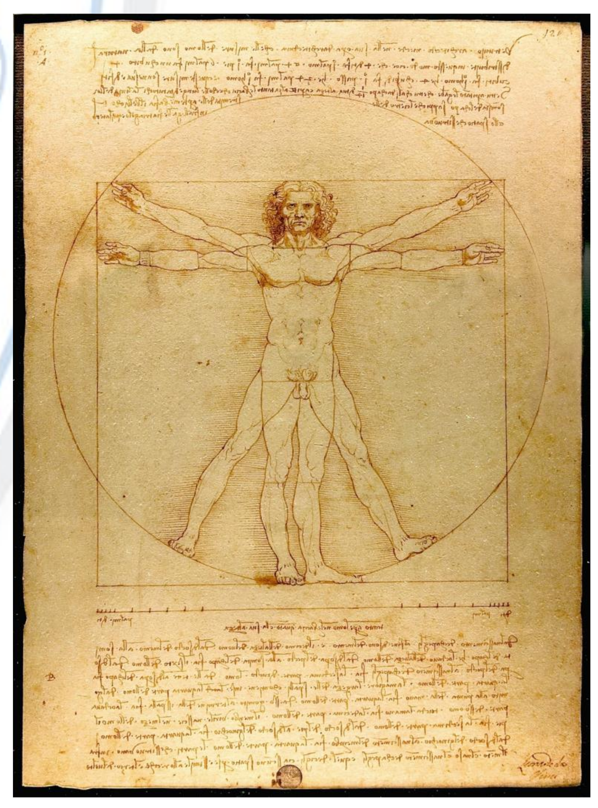
$$u=-K\mathbf{x}$$

$$K=[5.28 \quad -30.13 \quad 2.65 \quad -3.55]$$

Applications







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