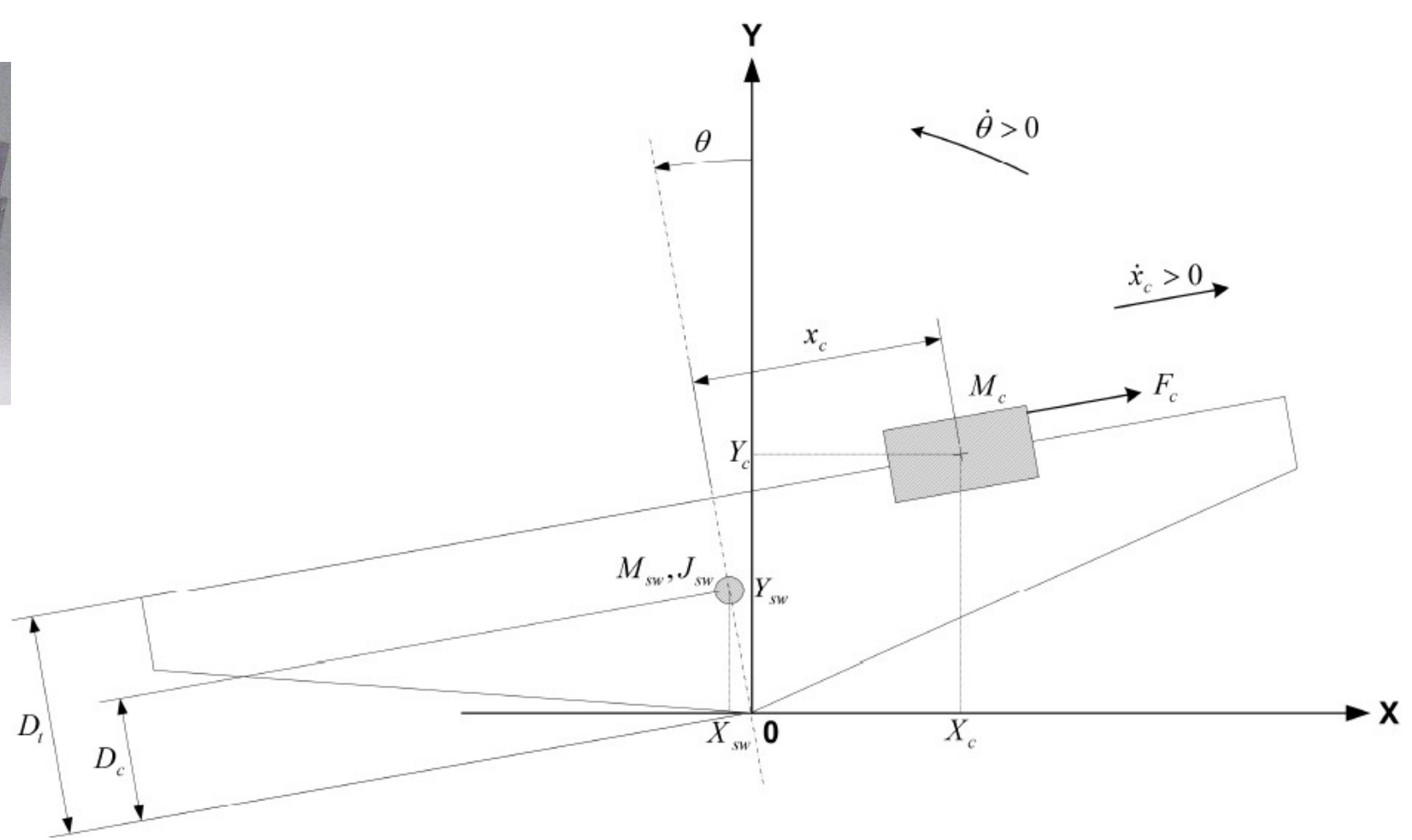
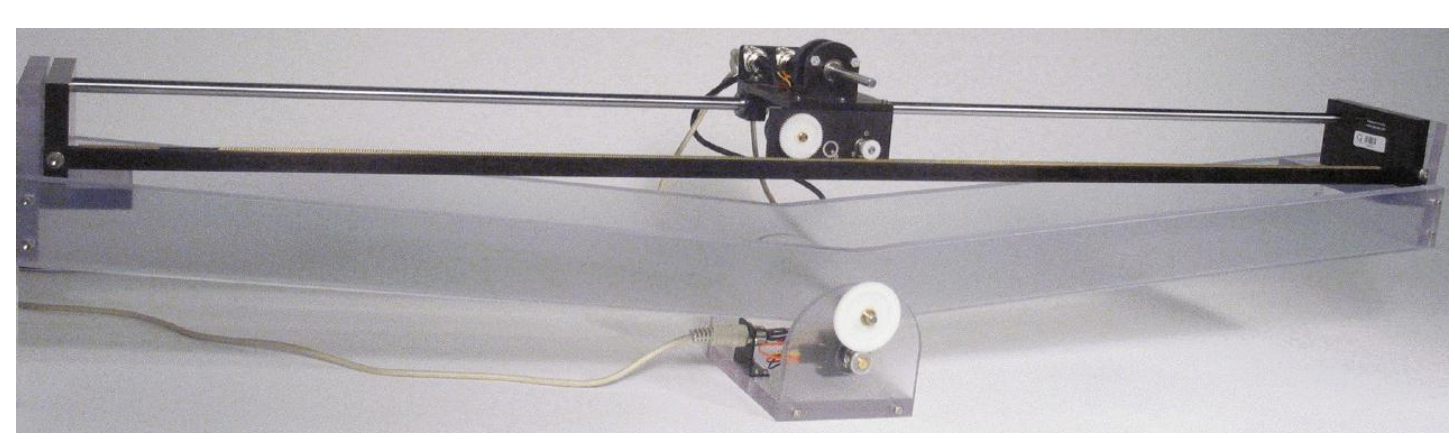
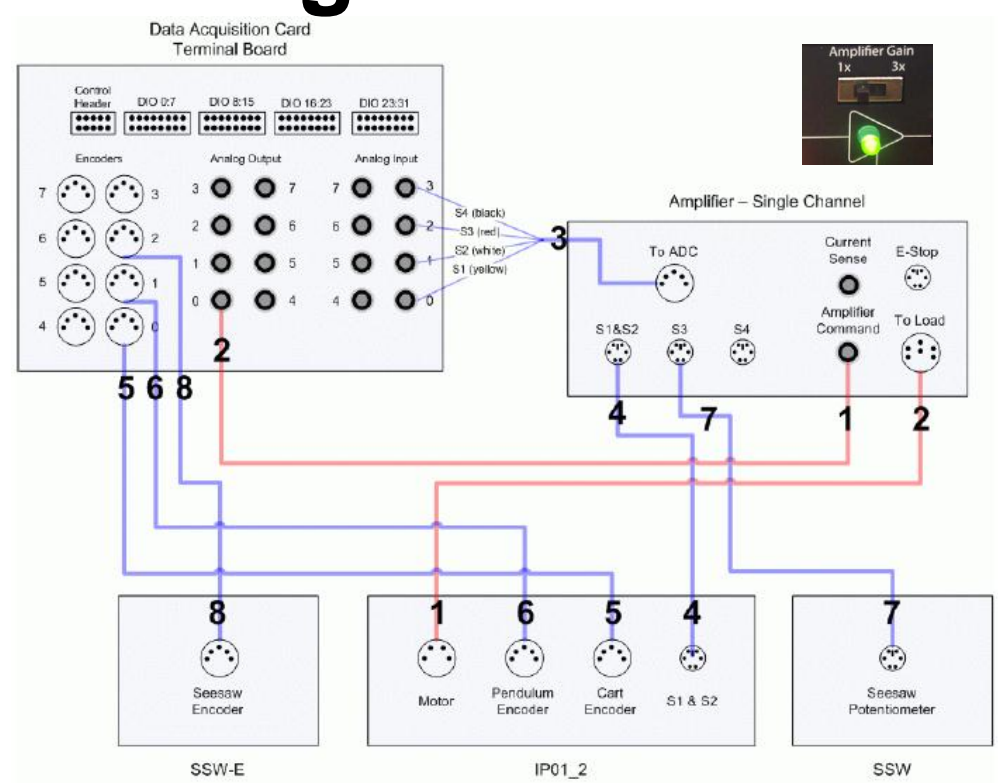


The Quanser Seesaw is an instrumented, fulcrum supported linear arm on which a rail mounted motorized cart device moves to balance the seesaw.



## Wiring schematics



## Requirements

Seesaw angular position:  $e_{ss}^{\delta} = 0$   $t_s^{5\%} \leq 4.5$   $\min V_m$   
 Cart position:  $t_s^{5\%} \leq 2.5$

## System dynamics

$$\ddot{x}_c = (M_c^2 x_c^3 \ddot{\theta}^2 + (F_c M_c - g M_c^2 \sin \theta - B_{eq} \dot{x}_c M_c)) x_c^2 - (2 M_c^2 D_t \dot{x}_c \dot{\theta} - (M_c^2 D_t^2 + M_c J_{sw})) \ddot{\theta}^2 + M_c^2 D_t g \cos \theta x_c - (M_c D_t^2 B_{eq} + B_{eq} J_{sw}) \dot{x}_c - M_c D_t B_{sw} \dot{\theta} + (F_c - g M_c \sin \theta) J_{sw} + M_c D_t^2 F_c + M_c D_t g M_{sw} D_c \sin \theta / (M_c^2 x_c^2 + J_{sw} M_c)$$

$$\ddot{\theta} = \frac{(D_t M_c \ddot{\theta}^2 - 2 M_c \dot{x}_c \dot{\theta} - g M_c \cos \theta) x_c + D_t F_c - D_t B_{eq} \dot{x}_c + g M_{sw} D_c \sin \theta - B_{sw} \dot{\theta}}{M_c \dot{x}_c + J_{sw}}$$

$$\ddot{x}_c = -\frac{M_c D_t g}{J_{sw}} x_c + \left( -g + \frac{D_t g M_{sw} D_c}{J_{sw}} \right) \theta - \left( \frac{D_t^2 B_{eq}}{J_{sw}} + \frac{B_{eq}}{M_c} \right) \dot{x}_c - \frac{D_t B_{sw}}{J_{sw}} \dot{\theta} + \frac{F_c}{M_c} + \frac{D_t^2 F_c}{J_{sw}}$$

$$\ddot{\theta} = -\frac{g M_c}{J_{sw}} x_c + \frac{g M_{sw} D_c}{J_{sw}} \theta - \frac{D_t B_{eq}}{J_{sw}} \dot{x}_c - \frac{B_{sw}}{J_{sw}} \dot{\theta} + \frac{D_t F_c}{J_{sw}}$$

$$\mathbf{x} = [x_c \quad \theta \quad \dot{x}_c \quad \dot{\theta} \quad \int \theta dt]^T$$

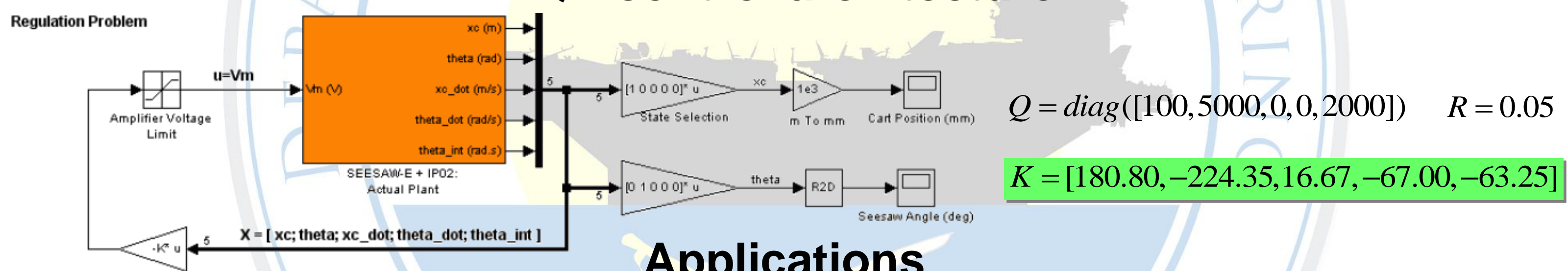
0	0	1	0	0	0
0	0	0	1	0	0
-1.77	-9.17	-21.6	0	0	3.10
-14.16	5.19	-3.81	0	0	0.55
0	1	0	0	0	0

## LQR controller design

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \quad J = \int_0^{\infty} (\mathbf{x}^T \mathbf{Q}\mathbf{x} + u \mathbf{R}u) dt$$

$$\mathbf{A}^T \mathbf{P} + \mathbf{P}\mathbf{A} - \mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T \mathbf{P} + \mathbf{Q} = 0 \quad \mathbf{K} = \mathbf{R}^{-1}\mathbf{B}^T \mathbf{P} \quad u = -\mathbf{K}\mathbf{x}$$

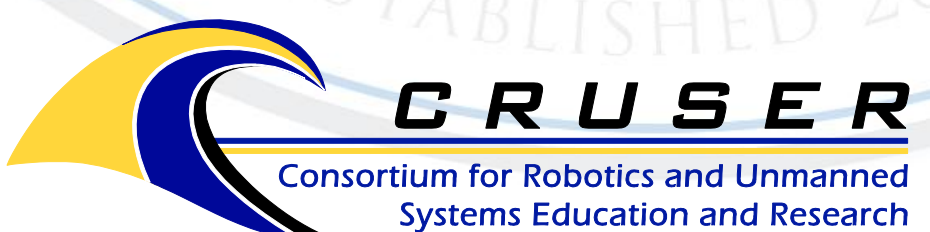
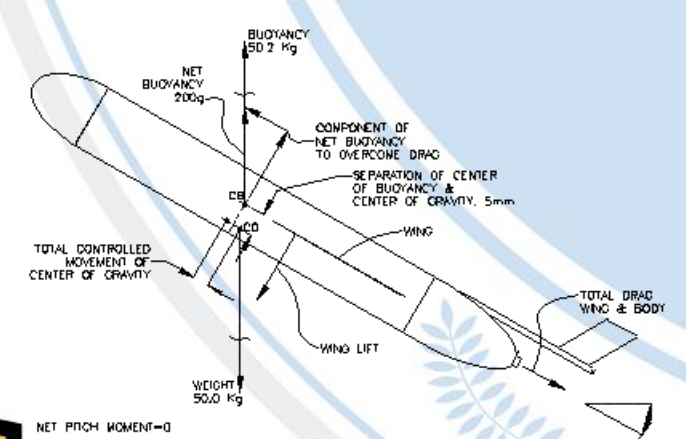
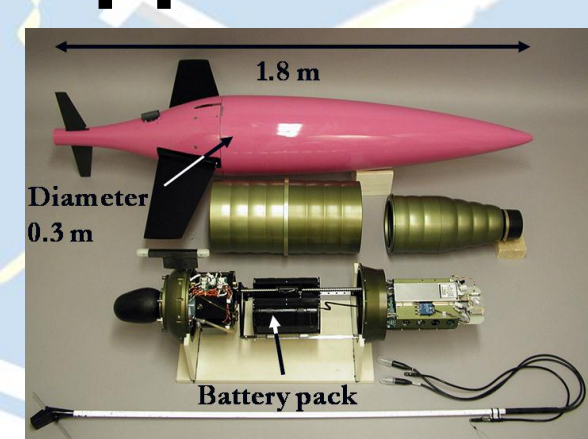
## LQR control architecture



$$\mathbf{Q} = \text{diag}([100, 5000, 0, 0, 2000]) \quad \mathbf{R} = 0.05$$

$$\mathbf{K} = [180.80, -224.35, 16.67, -67.00, -63.25]$$

## Applications



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