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Coordinated Path Following Control of Multiple Wheeled Robots with Directed Communication Links

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Abstract— **The paper addresses the problem of steering a fleet of wheeled robots along a set of given spatial paths, while keeping a desired inter-vehicle formation pattern. This problem arises for example when multiple vehicles are required to scan a given area in cooperation. In a possible mission scenario, one of the vehicles acts as a leader and follows a path accurately, while the other vehicles follow paths that are naturally determined by the formation pattern imposed. The solution adopted for coordinated path following builds on Lyapunov-based techniques and addresses explicitly the constraints imposed by the topology of the inter-vehicle communications network, which is captured in the framework of directed graph theory. With this set-up, path following (in space) and inter-vehicle coordination (in time) are essentially decoupled. Path following for each vehicle amounts to reducing a conveniently defined error variable to zero. Vehicle coordination is achieved by adjusting the speed of each of the vehicles along its path, according to information on the position of the other vehicles, as determined by the communications topology adopted. Simulations illustrate the efficacy of the solution proposed.**

Index Terms— **Coordinated Motion Control, Nonlinear Control, Path Following, Graph Theory, Wheeled Robots.**

I. INTRODUCTION

In recent years, there has been widespread interest in the problem of coordinated motion control of fleet of autonomous vehicles. Applications include aircraft and spacecraft formation flying control (Giuletti *et al.*, 2000), (Pratcher *et al.*, 2001), coordinated control of land robots (Ögren *et al.*, 2002) and control of multiple surface and underwater vehicles (Encarnação and Pascoal, 2001), (Lapierre *et al.*, 2003a), (Skjetne *et al.*, 2002).

At first inspection, the problem of coordinated motion control seems to fall within the domain of decentralized control. However, as clearly pointed out in (Fax and Murray, 2002), (Fax and Murray, 2003), it possesses several unique aspects that are at the root of new challenges to system designers. Among these, the following are worth stressing:

i) except for some cases in the area of aircraft control, the motion of one vehicle does not directly affect the motion of the other vehicles, that is, the vehicles are dynamically decoupled; the only coupling arises naturally out of the specification of the tasks that they are required to accomplish together.

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ii) there are strong practical limitations to the flow of information among vehicles, which is severely restricted by the nature of the supporting communications network. In marine robotics, for example, underwater communications rely on the propagation of acoustic waves. This sets tight limits on the communication bandwidths that are achievable. Thus, as a rule, possibly no vehicle will be able to communicate with the entire formation (Fax and Murray, 2003). A reliable vehicle coordination scheme should also exhibit some form of robustness against certain kinds of vehicle failures or loss of inter-vehicle communications.

A rigorous methodology to deal with some of the above issues has emerged from the work reported in (Fax and Murray, 2002), (Fax and Murray, 2003), which addresses explicitly the topics of information flow and cooperation control of vehicle formations simultaneously. The methodology proposed builds on an elegant framework that involves the concept of Graph Laplacian (a matrix representation of the graph associated with a given communication network). In particular, the results in (Fax and Murray, 2003) show clearly how the Graph Laplacian associated with a given inter-vehicle communication network plays a key role in assessing stability of the behaviour of the vehicles in a formation. It is however important to point out in that work that: i) the dynamics of the vehicles are assumed to be linear, time-invariant, and ii) the information exchanged among vehicles is restricted to linear combinations of the vehicles' state variables.

Inspired by the progress in the field, this paper tackles a problem in coordinated vehicle control that departs slightly from mainstream work reported in the literature. Specifically, we consider the problem of *coordinated path following* where *multiple vehicles are required to follow pre-specified spatial paths while keeping a desired inter-vehicle formation pattern in time*. This mission scenario occurs naturally in underwater robotics (Pascoal *et al.*, 2000). Namely, in the operation of multiple autonomous underwater vehicles for fast acoustic coverage of the seabed. In this important case, two or more vehicles are required to fly above the seabed at the same or different depths, along geometrically similar spatial paths, and map the seabed using copies of the same suite of acoustic sensors. By requesting that the vehicles traverse identical paths so as to make the acoustic beam coverage overlap along the seabed, large areas can be covered in a short time. This imposes constraints on the inter-vehicle formation pattern. Similar scenarios can of course be envisioned for land and air vehicles.

To the best of our knowledge, previous work on coordinated path following control has essentially been restricted to the area of marine robotics. See for example (Lapierre *et al.*, 2003a), (Lapierre *et al.*, 2003b), (Skjetne *et al.*, 2003) and the references therein. However, the solutions developed so far for *underactuated* vehicles are restricted to two vehicles in a leader-follower type of formation and lead to complex control laws. There is therefore a need to re-examine this problem to try and arrive at efficient and practical solutions.

A possible strategy is to consider similar problems for wheeled robots in the hope that the solutions derived for this simpler case may shed some light into the problem of coordinated path following for the more complex case of air and marine robots. Preliminary steps in this direction were taken in (Ghabcheloo *et al.*, 2004a), where the problem of coordinated path following of multiple wheeled robots was solved by resorting to linearization and gain scheduling techniques. The solutions obtained are conceptually simple and embody in themselves a straightforward mechanism that allows for the decoupling of path following (in space) and vehicle synchronization (in time). The price paid for the simplicity of the solutions is the lack of global results, that is, attractivity to so-called trimming paths and to a desired formation pattern can only be guaranteed locally, when the initial vehicle formation is sufficiently close to the desired one. The present paper overcomes this limitation and yields global results that allow for the consideration of arbitrary formation patterns and initial conditions. The solution adopted for coordinated path following builds on Lyapunov based techniques and addresses explicitly the constraints imposed by the topology of the inter-vehicle communications network. The latter is captured in the framework of directed graph theory, which allows for the consideration of communication topologies that contain *unidirectional links*. This is in striking contrast with some topologies studied in the literature where all links are bi-directional (that is, if vehicle i "talks" to vehicle j, then j "talks" to i as well). With the theoretical set-up adopted, path following (in space) and inter-vehicle coordination (in time) are essentially decoupled. Path following for each vehicle amounts to reducing a conveniently defined error variable to zero. Vehicle coordination is achieved by adjusting the speed of each of the vehicles along its path, according to information on the position of the other vehicles, as determined by the communications topology adopted. No other kinematic or dynamic information is exchanged among the robots. The coordination strategy is simple and holds great potential to be extended and applied to the case of air and marine robots.

The paper is organized as follows. Section II introduces the basic notation required, describes the simplified model of a wheeled robot, and offers a new solution to the problem of path following for a single vehicle. The main contribution of the paper is summarized in Section III, where a nonlinear control strategy for multiple vehicle cooperation is proposed. Section IV describes the results of simulations. Finally, Section V contains the main conclusions and presents problems that warrant further research.

II. PATH FOLLOWING

This section describes a new solution to the problem of path following control for a single wheeled robot. Due to space limitations, only the basic concepts and results will be presented. Complete details can be found in (Ghabcheloo *et al.*, 2004c) and (Kaminer *et al.*, 2005).

Consider a wheeled robot of the unicycle type depicted in Fig. 1, together a spatial path Γ to be followed. Two motorized rear wheels generate a control force and a control torque applied to the vehicle. The problem of path following can be briefly stated as follows:

Given a spatial path Γ, develop a feedback control law for the force and torque acting on a wheeled robot so that its center of mass converges asymptotically to the path while its total speed tracks a desired temporal profile.

An elegant solution to this problem was first advanced at a *kinematic* level in (Micaelli *et al.*, 1993), from which the following intuitive explanation is obtained: a path following controller should "look" at i) the distance from the vehicle to the path and ii) the angle between the vehicle's velocity vector and the tangent to the path, and reduce both to zero. This suggests that the kinematic model of the vehicle be derived with respect to a Serret-Frenet frame {T} that moves along the path, with {T} playing the role of the body-axis of a "virtual target vehicle" that must be tracked by the "real vehicle". Using this set-up, the abovementioned distance and angle become part of the coordinates of the error space where the path following control problem can be formulated and solved. The set-up adopted in (Micaelli *et al.*, 1993) was later reformulated in (Soeanto *et al.*, 2003), leading to a feedback control law that steers the *dynamic model* of a wheeled robot with parameter uncertainty along a desired path and yields global convergence results. This is in contrast with the results described in (Micaelli *et al.*, 1993), where only local convergence to the path has been proven. The key enabling idea involved in the derivation of a globally convergent path following control law is to add another degree of freedom to the rate of progression of a "virtual target" to be tracked along the path, thus bypassing the singularity problems that arise in (Micaelli *et al.*, 1993) when the position of the virtual target is simply defined by the projection of the actual vehicle on that path. Formally, this is done by making the center of the Serret-Frenet frame {T} that is attached to the path evolve according to an extra "virtual" control law.

To this effect, consider Fig. 1 where P is an arbitrary point on the path to be followed and Q is the center of the mass of the vehicle. Associated with P, consider the Serret-Frenet frame {T}. The signed curvilinear abscissa of P along the path is denoted by s. Let (x_e, y_e) denote the coordinates of Q in {T}. Further, let the rotations from {T} to {U} and from {B} to {U} be denoted by the yaw angles ψ_T and ψ_B, respectively. Define the variables v and $r = \dot{\psi}_B$ as the linear and angular speed of the robot, respectively, calculated in {U} and expressed in {B}. Simple calculations lead to the kinematics of the robot in the (x_e, y_e) coordinates as

$$\text{Kinematics} \begin{cases} \dot{x}_e &= (y_e c_c(s) - 1)\dot{s} + v\cos\psi_e \\ \dot{y}_e &= -x_e c_c(s)\dot{s} + v\sin\psi_e \\ \dot{\psi}_e &= r - c_c(s)\dot{s}. \end{cases} \quad (1)$$

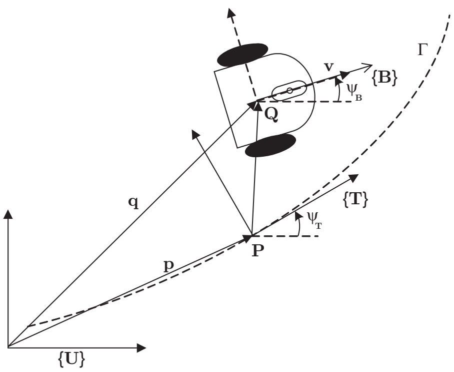

Fig. 1. Frames and error variables

where $\psi_e = \psi_B - \psi_T$ and $c_c(s)$ is the path curvature at P determined by s, that is $\dot{\psi}_T = c_c(s)\dot{s}$. Notice how the kinematics are driven by v, r and the term $\dot{s}$ that plays the role of an extra control parameter.

Under the usual simplifying assumptions, the dynamics of the wheeled robot can be written as

$$\text{Dynamics} \left\{ \begin{array}{l} \dot{v} = F/m \\ \dot{r} = N/I \end{array} \right. \tag{2}$$

where m denotes the mass of the robot, I is the moment of inertia about the vertical body-axis, and F and N denote the total force and torque, respectively. We assume without loss of generality that $m = I = 1$ in the appropriate units.

Driving the speed v to the desired speed is trivial to do with the simple control law $F = \dot{v_d} - k_0(v - v_d)$, which makes the error $v - v_d$ decay exponentially to zero. Controlling v is therefore decoupled from the control of the other variables, and all that remains is to find suitable control laws for N and for $\dot{s}$ to drive x_e, y_e, ψ_e to zero, no matter what the evolution of $v(t)$ is.

Proposition 1 [Path following]. *Let Γ be a path to be followed by a wheeled robot. Further let the kinematic and dynamic equations of motion of the robot be given by (1) and (2), respectively. Assume* $\lim_{t\to\infty} v(t) \neq 0$. *Define*

$$\begin{array}{l} \sigma = \sigma(y_e) = -\text{sign}(v)\sin^{-1} k_2 \frac{y_e}{|y_e|+\epsilon_0} \\ \Delta(\psi_e, \sigma) = \left\{ \begin{array}{ll} 1 & \text{if} \psi_e = \sigma \\ \frac{\sin\psi_e - \sin\sigma}{\psi_e - \sigma} & \text{otherwise} \end{array} \right. \\ \phi = c_c\dot{s} + \dot{\sigma} - k_1(\psi_e - \sigma) - v y_e \Delta(\psi_e, \sigma) \end{array}$$

for some $k_1 > 0$, $0 < k_2 \leq 1$ and $\epsilon_0 > 0$. Let the control laws for N and $\dot{s}$ be given by

$$N = \dot{\phi} - k_4(r - \phi) - (\psi_e - \sigma) \tag{3}$$

$$\dot{s} = v\cos\psi_e + k_3 x_e \tag{4}$$

for some $k_3, k_4 > 0$. Then, x_e, y_e, and ψ_e are driven asymptotically to zero from any initial condition.

Indication of proof. At a purely kinematic level, consider the Lyapunov function candidate,

$$V = \frac{1}{2}x_e^2 + \frac{1}{2}y_e^2 + \frac{1}{2}(\psi_e - \sigma)^2 \tag{5}$$

which is positive definite and radially unbounded. With $r = \phi$ and $\dot{s}$ as in (4), the time derivative of V along the trajectories of (1) yields

$$\dot{V} = -k_3 x_e^2 - k_1(\psi_e - \sigma)^2 - k_2|v|\frac{y_e^2}{|y_e| + \epsilon_0} \tag{6}$$

which is negative definite under the given condition on the velocity v. Standard back-stepping technique can now be used to include the dynamics and to show that with the control law (3) for torque N, the states x_e, y_e, ψ_e are driven to zero. Notice that $\dot{s}$ tends to v, that is, the speed of the virtual target approaches v asymptotically.

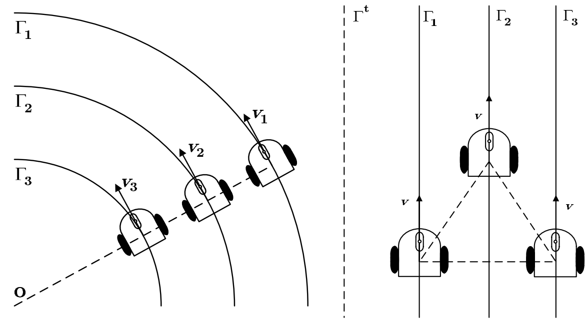

Fig. 2. Coordination: in-line and triangle formations

III. COORDINATION

Equipped with the results obtained in the previous section, we now consider the problem of coordinated path following control that is the main contribution of the present paper. In the most general set-up, one is given a set of $n \geq 2$ wheeled robots and a set of n spatial paths Γ_k; $k = 1, 2, ..., n$ and require that robot k follow path Γ_k. We further require that the vehicles move along the paths in such a way as to maintain a desired formation pattern compatible with those paths. Figure 2 shows the simple cases where 3 vehicles are required to follow straight paths or circumferences $\Gamma_i; i = 1, 2, 3$ while keeping a desired "triangle" or "in-line" formation pattern, respectively.

In the simplest case, the paths Γ_i may be obtained as simple parallel translations of a "template" path Γ^t (Fig. 2, right). A set of paths can also be obtained by considering the case of scaled circumferences with a common center and different radii R_i (Fig. 2, left). In this paper, for simplicity of presentation, we restrict ourselves to "in-line" formation patterns and to the types of paths described above. However, the methodology proposed for coordinated path following can be extended to deal with arbitrary paths in the case of bidirectional communication links (Ghabcheloo *et al.*, 2004c).

Assuming that separate path following controllers have been implemented for each robot, it now remains to coordinate (that is, synchronize) them in time so as to achieve a desired formation pattern. As will become clear, this will be achieved by adjusting the speeds of the robots as functions of the "along-path" distances among them. Consider for example the case of in-line formations maneuvering along parallel translations of straight lines as in Fig. 3. For robot i, let s_i denote the signed curvilinear abcissa of the origin of the corresponding Serret-Frenet frame $\{T_i\}$ being tracked. Since each vehicle body-frame $\{B_i\}$ tends asymptotically to $\{T_i\}$,

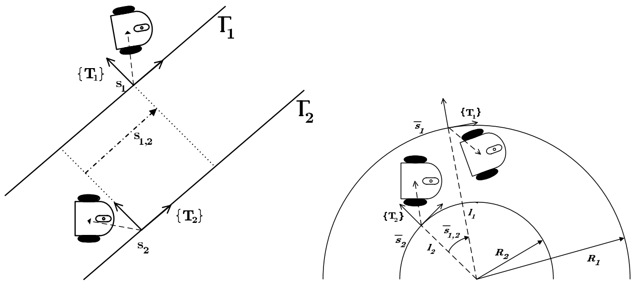

Fig. 3. Along-path distances: straight lines and circumferences

it follows that the vehicles are (asymptotically) synchronized if $s_{i,j}(t) := s_i(t) - s_j(t) \rightarrow 0, t \rightarrow \infty; i = 1, .., n$. This shows that in the case of translated straight lines $s_{i,j}$ is a good measure of the along-path distances among the robots. Similarly, in the case of scaled circumferences an appropriate measure of the distances among the robots is $\bar{s}_{i,j} := \bar{s}_i - \bar{s}_j; i = 1, .., n$, where $\bar{s}_i = s_i/R_i$. See Fig. 3.

Notice how the definition of $\bar{s}_{i,j}$ relies on a normalization of the lengths of the circumferences involved and is equivalent to computing the angle between vectors l_i and l_j directed from the center of the circumferences to origin of the Serret-Frenet frames $\{\mathrm{T}_i\}$ and $\{\mathrm{T}_j\}$, respectively. In both cases, we say that the vehicles are coordinated if the corresponding along path distance is zero, that is, $s_i - s_j = 0$ or $\bar{s}_i - \bar{s}_j = 0$. The extension of these concepts to a more general setting requires that each path Γ_i be parameterized in terms of a parameter ξ_i that is not necessarily the arc length along the path. An adequate choice of the parameterization will allow for the conclusion that the vehicles are synchronized iff $\xi_i = \xi_j$ for all i, j. For example, in the case of two robots following two circumferences with radii R_1 and R_2 while keeping an in-line formation pattern, $\xi_i = s_i/R_i; i = 1, 2$.

In what follows, we start by computing the coordination error dynamics, after which a decentralized feedback control law is derived to drive the coordination error to zero asymptotically. In the analysis, the theory of directed graphs - as the mathematical machinery par excellence to deal with inter-vehicle communication constraints - will play a key role.

A. Coordination error dynamics

As explained before, we let path Γ_i be parameterized by ξ_i and denote by $s_i = s_i(\xi_i)$ the corresponding arc length. We further define $R_i(\xi_i) = \frac{\partial s_i}{\partial \xi_i}$ and assume $R_i(\xi_i)$ is positive and bounded for all ξ_i. The symbol $R_i(.)$ is motivated by the nomenclature adopted for the case of paths that are nested arcs of circumferences. Using equation (4), it is straightforward to show that the evolution of ξ_i is given by $\dot{\xi}_i = (v_i \cos\psi_{ei} + k_{3i} x_{ei})/R_i(\xi_i)$, which can be written as

$$\dot{\xi}_i = \frac{1}{R_i(\xi_i)} v_i + d_i \tag{7}$$

where $d_i = \frac{1}{R_i(\xi_i)}(\cos\psi_{ei} - 1)v_i + k_{3i} x_{ei}$. Notice from the previous section that $d_i \rightarrow 0$ asymptotically as $t \rightarrow \infty$. Suppose one vehicle, henceforth referred to as vehicle $\mathcal{L}$, is elected as a "leader" and let the corresponding path $\Gamma_{\mathcal{L}}$ be parametrized by its length, that is, $\xi_{\mathcal{L}} = s_{\mathcal{L}}$. In this case, $R_{\mathcal{L}}(\xi_{\mathcal{L}}) = 1$. It is important to point out that $\mathcal{L}$ can always be taken as a "virtual" vehicle that is added to the set of "real" vehicles as an expedient to simplify the coordination strategy. Let $v_{\mathcal{L}} = v_{\mathcal{L}}(t)$ be a desired speed profile assigned to the leader in advance, and known to all the other vehicles. Notice now that in the ideal steady situation where the vehicles move along their respective paths while keeping the desired formation, we have $\xi_i - \xi_{\mathcal{L}} = 0; i = 1, .., n$. From (7), making $d_i = 0$, it follows that the desired velocities of vehicles $1 \leq i \leq n$ equal $R_i(\xi_i)v_{\mathcal{L}}(t)$. This suggests the introduction of the speed-tracking error variable

$$\eta_i = v_i - R_i(\xi_i)v_{\mathcal{L}}, \;\; 1 \leq i \leq n \tag{8}$$

Taking into account the vehicle dynamics yields

$$\dot{\eta}_i = u_i = F_i - \frac{d}{dt}\left(R_i(\xi_i)v_{\mathcal{L}}\right). \tag{9}$$

It is also easy to compute the dynamics of the origin of each Serret-Ferret frame $\{\mathrm{T}_i\}$ as

$$\dot{\xi}_i \;\; = \;\; \tfrac{1}{R_i}\eta_i + v_{\mathcal{L}} + d_i \tag{10}$$

To write the above dynamic equations in vector form, define $\eta = [\eta_i]_{n\times 1}$, $\xi = [\xi_i]_{n\times 1}$, $u = [u_i]_{n\times 1}$, $d = [d_i]_{n\times 1}$ and $C = C(\xi) = \mathrm{diag}[1/R_i(\xi_i)]_{n\times n}$ to obtain

$$\begin{aligned} \dot{\eta} &= u \\ \dot{\xi} &= C\eta + v_{\mathcal{L}}\mathbf{1} + d \end{aligned} \tag{11}$$

where $\mathbf{1} = [1]_{n\times 1}$. Note that $||d|| \rightarrow 0$ asymptotically as $t \rightarrow \infty$ and C is positive definite and bounded. Clearly, the objective is to derive a control strategy for u to make $\xi_1 = ... = \xi_n$. This calls for the definition of an appropriate error space where the problem of coordinated motion control can be naturally formulated. To do this, it is important to address the constraints that the inter-vehicle communication system imposes on the structure of the possible types of control laws.

The general form of feedback control law that we consider for each vehicle can be written as $u_i = u_i(\eta_i, \xi_j : j \in J_i)$ where J_i is the index set that determines what path parameters $\xi_j : j \neq i$ are transmitted to vehicle i. With this control law, each vehicle i requires only access to its own speed and path parameter and to some or all of the path parameters of the remaining vehicles, as defined by the index set J_i. Clearly, the index sets capture the type of communication structure that is available for vehicle coordination. It is thus natural that the machinery of directed graph theory be brought to bear on the definition of the problem under study. The constraints of the problem at hand require that we consider directed graphs in order to address the case where the communication channels among the vehicles include unidirectional links. At this point, and before we proceed, we recall some properties of graph theory and matrices, (Bang-Jensen, 2002), (Godsil and Royle, 2001), and (Horn, 1985).

A directed graph or digraph $\mathcal{G} = \mathcal{G}(\mathcal{V}, \mathcal{E})$ consists of a finite set $\mathcal{V}$ of vertices $V_i \in \mathcal{V}$ and a finite set $\mathcal{E}$ of ordered pairs $(V_i, V_j) \in \mathcal{E}$, henceforth referred to as arcs. Given an arc $(V_i, V_j) \in \mathcal{E}$, its first and second elements are called the tail and head of the arc, respectively. In the present work, the vertices of a graph represent vehicles and the arcs the directional data links among them. Following standard notation, the flow of information in an arc is always directed from its head to its tail. The in-degree (out-degree) of a vertex V_i is the number of arcs with V_i as its head (tail). If $(V_i, V_j) \in \mathcal{E}$, then we say that V_i and V_j are adjacent.

A path of length N from V_i to V_j in a digraph is a sequence of $N+1$ distinct vertices starting with V_i and ending with V_j such that consecutive vertices are adjacent. If there is a path in $\mathcal{G}$ from vertex V_i to vertex V_j, then V_j is said to be reachable from V_i. In this case, there is a path of consecutive communication links directed from vehicle j (transmitter) to vehicle i (receiver). A vertex V_i is globally reachable if it is reachable from every other vertex.

The adjacency matrix of a digraph $\mathcal{G}$, denoted A, is a square matrix with rows and columns indexed by the vertices, such that the i,j-entry of A is 1 if $(V_i, V_j) \in \mathcal{E}$ and zero otherwise. The degree matrix D of a digraph $\mathcal{G}$ is a diagonal matrix where the i,i-entry equals the *out-degree* of vertex V_i. The Laplacian of a digraph is defined as $L = D - A$. The following Lemma plays a key role in the development that follows.

Lemma (Lin, 2005): The Laplacian matrix of a digraph with at least one globally reachable vertex has a simple eigenvalue at 0 with right eigenvector $\mathbf{1}$. All the other eigenvalues have positive real parts.

From the lemma, if L is the Laplacian of a digraph with a globally reachable vertex, then there exists a nonsingular matrix F, partitioned as

$$F = \begin{pmatrix} \mathbf{1} & F_1 \end{pmatrix},\ F^{-1} = \begin{pmatrix} \beta^T \\ F_2 \end{pmatrix}, \tag{12}$$

such that

$$F^{-1}LF = \begin{pmatrix} 0 & \mathbf{0} \\ \mathbf{0}^T & L_{11} \end{pmatrix}, \tag{13}$$

where $\mathbf{0} = [0, ..., 0]^T$ is a vector of appropriated dimensions, β is a left eigenvector associated with the 0 eigenvalue, and L_{11} is nonsingular and has the property that all its eigenvalues have positive real parts. From the fact that $F^{-1}F = I_n$ it can be easily concluded that

$$F_2F_1 = I_{n-1},\ F_2\mathbf{1} = \mathbf{0},\ \beta^T F_1 = \mathbf{0}^T,\ L = F_1L_{11}F_2. \tag{14}$$

Define $\theta = F_2\xi \in \mathbb{R}^{n-1}$. From (11), assuming that the inter-vehicle communication network is described by a directed graph with at least one globally reachable vertex, it follows that the dynamics of (η, θ) can be written as

$$\begin{array}{rcl} \dot{\eta} & = & u \\ \dot{\theta} & = & F_2C\eta + F_2d. \end{array} \tag{15}$$

Since $F_2\mathbf{1} = \mathbf{0}$ and $\text{rank}F_2 = n-1$, then $\theta = \mathbf{0} \Leftrightarrow \xi_i = \xi_j$ for all i, j. Consequently, if θ is driven to zero asymptotically, so are the coordination errors $\xi_i - \xi_j$ and the problem of coordinated path following is solved.

Coordination Problem formulation. *Consider the coordination system with dynamics (15), where d is a bounded signal that tends asymptotically to zero. Assume each vehicle has access to its own state and exchanges information on its path parameter ξ with some or all of the companion vehicles, as determined by the constraints of the underlying communications network (defined by the Laplacian matrix L or index sets J_i). Determine a feedback control law for u that will drive θ and η asymptotically to zero.*

Note. This paper tackles the case where the matrix $C = C(\xi) = \text{diag}[1/R_i(\xi_i)]$ is constant. While restrictive, it still allows to consider paths that consist of straight lines, nested circumferences, and parallel translations of one arbitrary path. The main result on coordination control is stated below.

B. Coordination control

Proposition 2 [Coordination]. *Consider the coordination dynamics (11) with constant matrix C. The control law*

$$u = -A\eta - BC^{-1}L\xi, \tag{16}$$

solves the coordination problem for positive definite scalar matrices $A = aI$ and $B = bI$ if the graph $\mathcal{G}(L)$ has at least one globally reachable vertex and

$$\frac{a^2}{b} > \max_{0 \neq \mu \in \sigma(L)} \left(\frac{\text{Im}(\mu)^2}{\text{Re}(\mu)}\right), \tag{17}$$

where $\sigma(.)$ stands for the spectrum of a matrix and $\text{Im}(.)$ and $\text{Re}(.)$ denote the imaginary and real part of a complex number, respectively. Furthermore, the control law (16) can be written in decentralized form as

$$u_i = -a\,\eta_i - bR_i(\xi_i)\sum_{j \in J_i}(\xi_i - \xi_j). \tag{18}$$

Proof. To solve the coordination problem, consider for the time being the case where $d = \mathbf{0}$ (this assumption will be lifted later). The unforced closed-loop coordination system consisting of equations (15) and (16) can be written as

$$\begin{array}{rcl} \dot{\eta} & = & -A\eta - BC^{-1}F_1L_{11}\theta \\ \dot{\theta} & = & F_2C\eta. \end{array} \tag{19}$$

We show that the closed-loop matrix

$$A_{cl} = \begin{pmatrix} -A & -BC^{-1}F_1L_{11} \\ +F_2C & 0 \end{pmatrix} \tag{20}$$

is a stability (Hurwitz) matrix. Using a similary transformation $\text{diag}(I, CF)$, it can be shown that $\lambda = -a$ is an eigenvalue of A_{cl} and the rest of the eigenvalues are given by the roots of $\det(\lambda(\lambda + a)I + bL_{11}) = 0$. Let $\mu \in \sigma(L_{11})$ with $\text{Re}(\mu) > 0$ be an arbitrary eigenvalue of L_{11}. From the last equation it follows that $\mu = -\lambda(\lambda + a)/b$. Clearly, to every μ there correspond two possible eigenvalues which have negative real part if a and b satisfy (17).

Consider now the situation where the closed loop system is driven by the exogenous signal $d \neq \mathbf{0}$. Since the unforced system is globally uniformly exponentially stable (GUES), the new system is input-to-state stable (ISS). Furthermore, since $d(t) \to 0$ asymptotically as $t \to \infty$, so do η and θ, thus completing the proof of the theorem. See (Sontag, 1996) and (Khalil, 2000) pp. 174-176 for details. Together with the result on path following control in Section II, the above theorem yields a methodology for coordinated path following control of multiple wheeled robots. This statement follows from the fact that, with the control laws derived, the conditions under which path following is achieved are indeed verified. Namely, the velocity $v(t)$ of each robot remains bounded and $\lim_{t \to \infty} v(t) \neq 0$ (Ghabcheloo *et al.*, 2004c).

Remark. Notice that the eigenvalues of the closed-loop coordination system are independent of matrix C. A similar result was derived in (Ghabcheloo *et al.*, 2004a) for a simple communication structure, using a local linearization approach. The present result is therefore a non-trivial extension of the results in (Ghabcheloo *et al.*, 2004a) to a nonlinear setting and to more realistic communication structures.

IV. Simulations

This section contains the results of simulations that illustrate the performance obtained with the control laws developed. Figures 4 and 5 correspond to a simulation where 3 wheeled robots were required to follow 3 circumferences, with radii 2[m], 3[m] and 4[m], while keeping an in-line formation pattern. The Laplacian matrix L (that captures the communication network among the vehicles) and the coordination controller gains are

$$L = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{pmatrix} \text{and} \quad a = 2; b = 1, \qquad (21)$$

respectively. The eigenvalues of L belong to the set $\{0, 0.86 \pm j0.5\}$, thus verifying condition (17). A virtual leader vehicle was defined and its speed set to a constant value of 0.1[m/s]. Figure 4 (left) shows the evolution of the vehicles as they start from the same point off the assigned paths and converge to the latter. Figure 4 (right) is a plot of the vehicle speeds that ensure coordination along the paths. Finally, Figure 5 shows the coordination errors $\xi_1 - \xi_2$ and $\xi_1 - \xi_3$ and the path following errors y_{ei} decaying to 0.

V. Conclusions and suggestions for further research

The paper formulated and presented a solution to the problem of steering a fleet of wheeled robots along a set of given spatial paths, while keeping a desired inter-vehicle formation pattern. With the set-up adopted, path following (in space) and inter-vehicle coordination (in time) are essentially decoupled. Path following for each vehicle amounts to reducing a conveniently defined error vector to zero. Vehicle coordination is achieved by adjusting the speed of each of the vehicles along its path, according to information on the position of the other vehicles, as determined by the communications topology adopted. The methodology proposed led to a decentralized control law whereby the exchange of data among the vehicles is kept at a minimum and is channeled through communication links that are not necessarily bidirectional. Simulations illustrated the efficacy of the solution proposed. Further work is required to extend the methodology proposed to air and underwater vehicles.

References

Bang-Jensen, J. and Gutin, G. (2002). *Digraph Theory, Algorithms and Applications*, Springer.

Encarnação, P., and Pascoal, A. (2001). Combined trajectory tracking and path following: an application to the coordinated control of marine craft. *IEEE Conf. Decision and Control (CDC'2001)*,Orlando, Florida.

Fax, A. and Murray, R. (2002a). Information Flow and Cooperative Control of Vehicle Formations. *Proc. 2002 IFAC*, Barcelona, Spain.

Fax, A. and Murray, R. (2002b). Graph Laplacians and Stabilization of Vehicle Formations In : *Proc. 2002 IFAC*, Barcelona, Spain.

Ghabcheloo, R., Pascoal, A., Silvestre, C., and Kaminer, I. (2004a). Coordinated Path Following Control of Multiple Wheeled Robots. *Proc. 5th IFAC Symposium on IAV*, July 5-7, 2004. Lisbon, Portugal.

Ghabcheloo, R., Pascoal, A., and Silvestre, C. (2004b).*Coordinated Path Following Control using Linearization Techniques*. Internal Report CPF01, Institute for Systems and Robotics, Jan. 2004.

Ghabcheloo, R., Pascoal, A., and Silvestre, C. (2004c). *Coordinated Path Following Control using Nonlinear Techniques*. Internal Report CPF02, Institute for Systems and Robotics, Nov. 2004.

Giuletti, F., Pollini, L., and Innocenti, M. (2000). Autonomous formation flight. *IEEE Control Systems Magazine*, vol. 20, pp. 34 - 44.

Godsil, C. and Royle, G. (2001). *Algebraic Graph Theory*. Graduated Texts in Mathematics, Springer-Verlag New York, Inc.

Horn, R.A. and Johnson, C.R. (1985). *Matrix Analysis*, Cambridge Univ.

Khalil, H. K. (2000). *Nonlinear Systems*. Third Edition, Pearson Eduacation International, Inc. New Jersey.

Kaminer, I., Pascoal, A. and Yakimenko, O. (2005). Nonlinear Path Following Control of Fully Actuated Marine Vehicles with Parameter Uncertainty, *16th IFAC World Congress*, Prague, Czech Republic.

Lapierre, L., Soetanto, D., and Pascoal, A. (2003). Coordinated motion control of marine robots. *Proc. 6th IFAC Conference on Manoeuvering and Control of Marine Craft (MCMC2003)*, Girona, Spain.

Lapierre, L., Soetanto, D. and Pascoal, A. (2003). Nonlinear path following control of autonomous underwater vehicles. *Proc. 1st IFAC Conference on GCUV*, Newport, South Wales, UK.

Lin Z., Francis, B. and Maggiore, M. (2005). Necessary and Sufficient Graphical Conditions for Formation Control of Unicycles. *IEEE Trans. on Automatic Control*, Vol. 50, No. 1, 2005, pp. 121-127

Micaelli, A. and Samson, C. (1993). Trajectory - tracking for unicycle - type and two - steering - wheels mobile robots. *Technical Report No. 2097*. INRIA, Sophia-Antipolis, France.

Ögren, P., Egerstedt, M., and Hu, X. (2002). A control lyapunov function approach to multiagent coordination. *IEEE Trans. on Robotics and Automation*, Vol. 18.

Pascoal, A. et al. (2000). Robotic ocean vehicles for marine science applications: the european ASIMOV Project. *Proc. OCEANS'2000 MTS/IEEE*, Rhode Island, Providence, USA.

Pratcher, M., D'Azzo, J., and Proud, A. (2001). Tight formation control. *Journal of Guidance, Control and Dynamics*, Vol. 24, N0. 2, March-April 2001, pp. 246-254.

Rouche, N., Habets, P. and Laloy, M. (1993). *Stability theory by Liapunov's direct method*. Springer-Verlag New York, Inc.

Skjetne, R., Moi, S., and Fossen, T. (2002). Nonlinear formation control of marine craft. *Proc. IEEE CDC*, Las Vegas, NV.

Skjetne, R., Flakstad, I., and Fossen, T. (2003). Formation control by synchronizing multiple maneuvering systems. *Proc. 6th IFAC Conf. on Manoeuvering and Control of Marine Craft (MCMC03)*, Girona, Spain.

Soetanto, D., Lapierre, L., and Pascoal, A. (2003). Adaptive, Non-Singular Path Following, Control of Dynamic Wheeled Robots *Proc. ICAR'03*, Coimbra, Portugal.

Sontag E. D. and Wang, Y.(Sept. 1996). New characterizations of input-to-state stability. *IEEE Trans. on Aut. Cont.*, Vol.41 Issue 9., pp 1283-1294

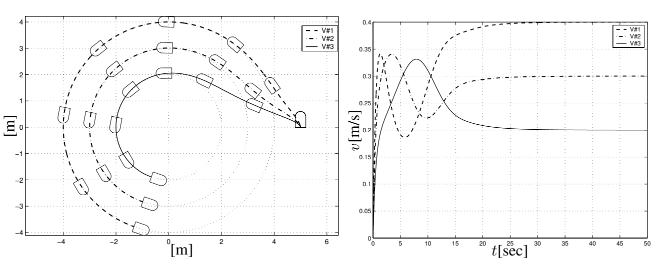

Fig. 4. Coordination of 3 vehicles. Left: Vehicle paths. Right: Speeds.

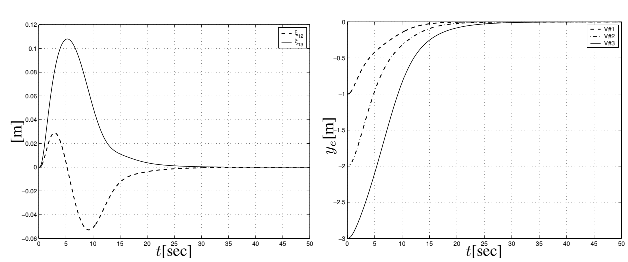

Fig. 5. Left: Coordination errors. Right: Path following errors.