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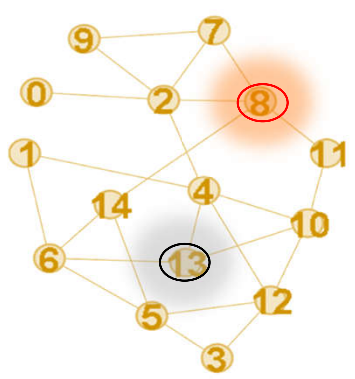

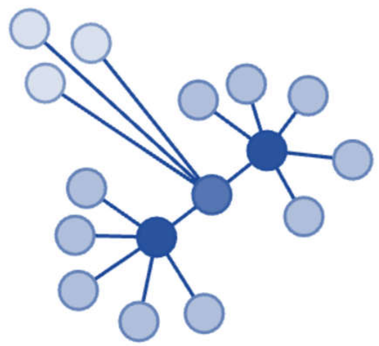

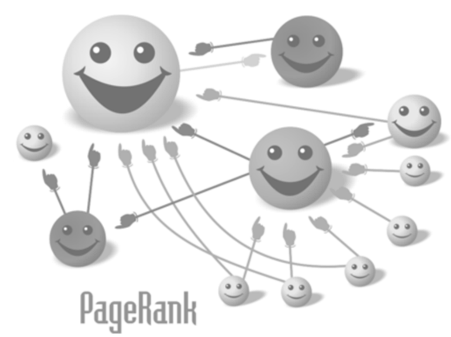

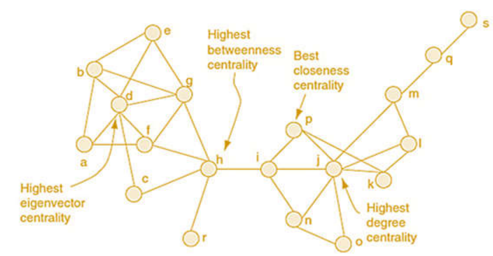

MA4404 Complex Networks

Closeness Centrality

Learning Outcomes

- Understand the new categories of centralities that includes closeness centrality.
- Compute Closeness Centrality per node.
- Interpret the meaning of the values of Closeness Centrality.

Recall…back to Degree Centrality

Quality: what makes a node important (central)	Mathematical Description	Appropriate Usage	Identification
Lots of one-hop connections from v	The number of vertices that v influences directly	Local influence matters Small diameter	Degree centrality (or simply the $\deg(v)$)
Lots of one-hop connections from v relative to the size of the graph	The proportion of the vertices that v influences directly	Local influence matters Small diameter	Normalized degree centrality $\frac{\deg(v)}{\lvert V(G) \rvert}$
In the "middle" of the graph	HOW?		

How to measure it?

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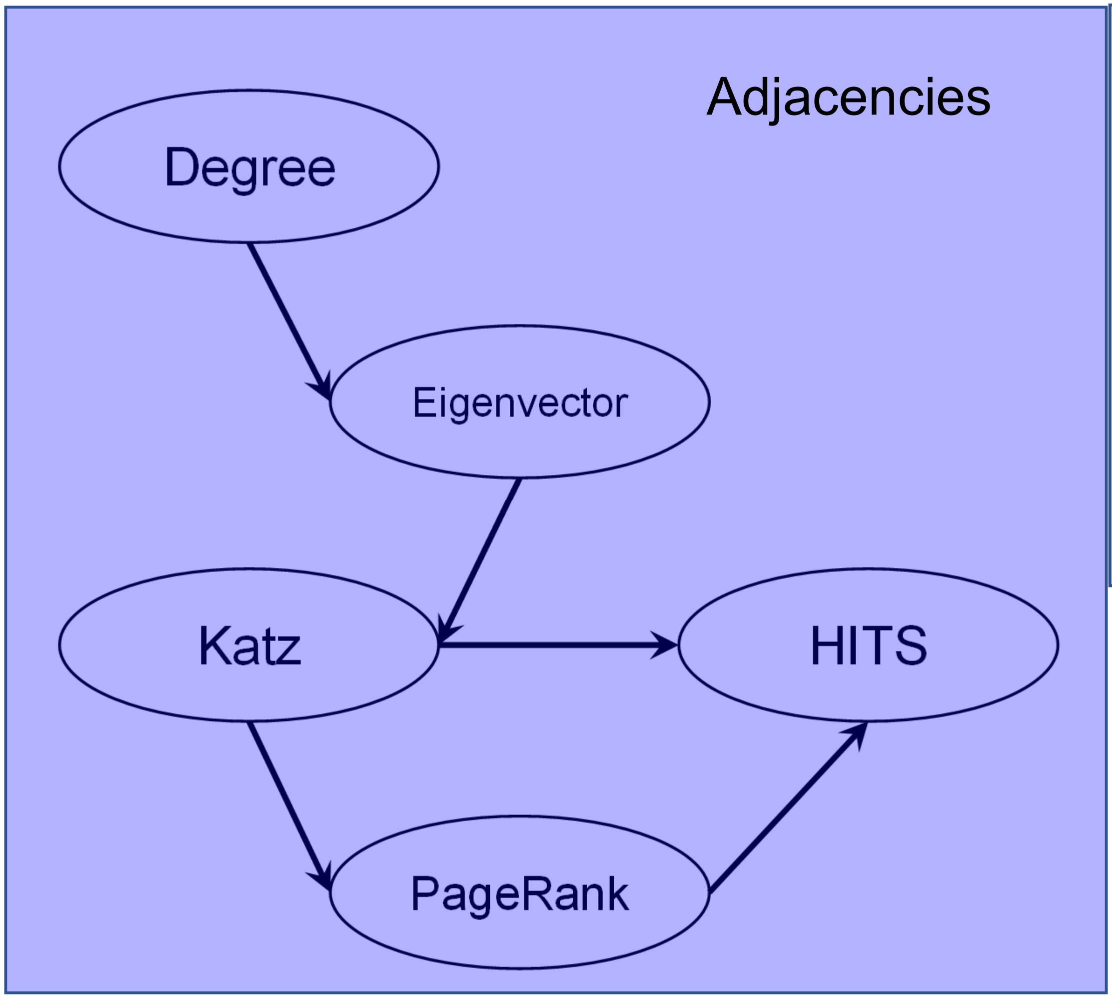
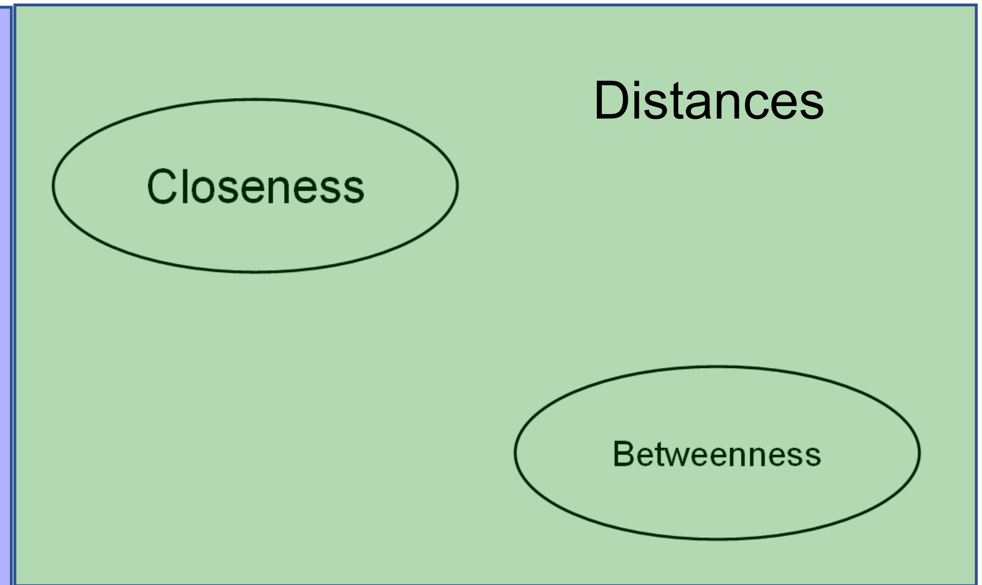
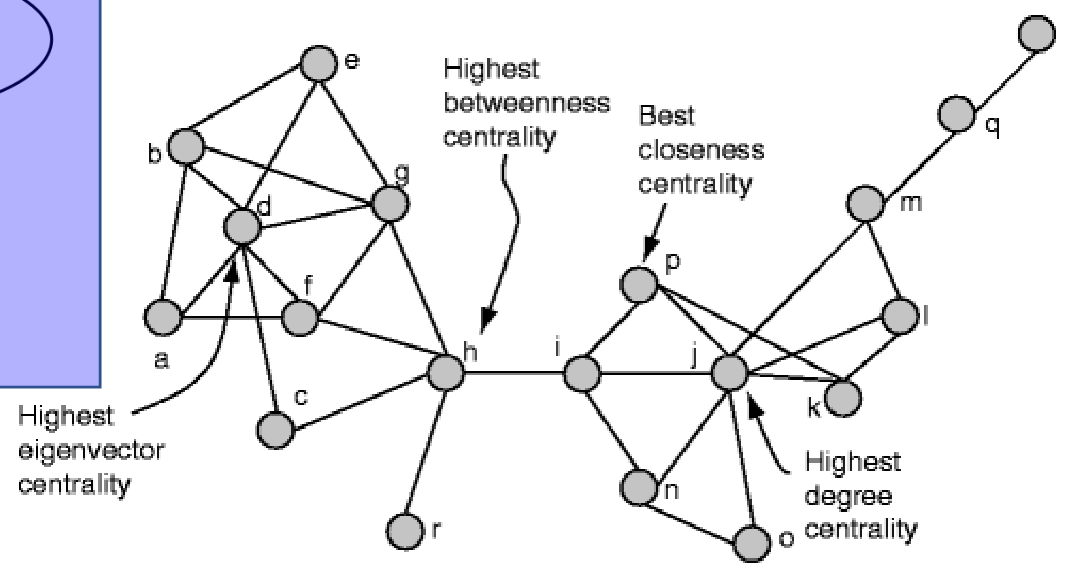
In the "middle" of the graph - closeness centrality	Close to everyone at the same time	The efficiency of a vertex of reaching everyone quickly (spreading news or a virus for example)	$C_i = 1 \Big/ \sum_{j=1}^{n} d(i,j)$

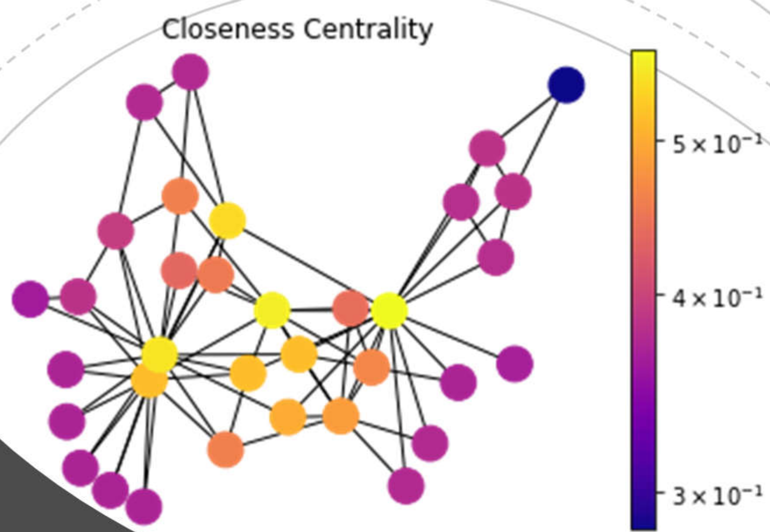
MA4404: Centralities categories







[Discovering Sets of Key Players in Social Networks | SpringerLink]



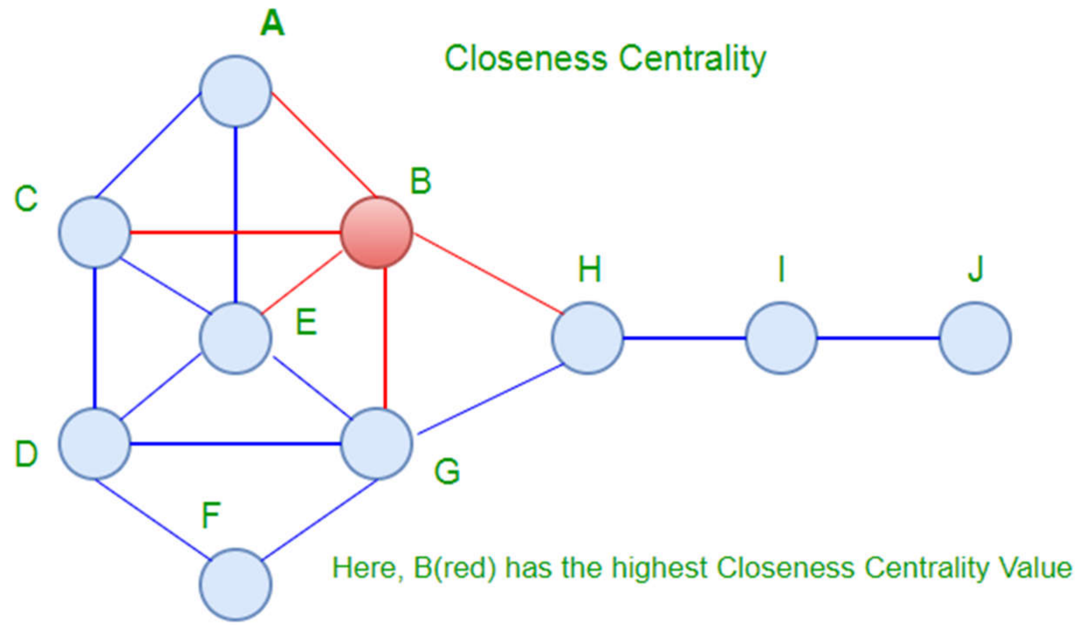
Intuition Closeness Centrality

Why?!

- What if it’s not so important to have many direct friends?
- But one still wants to be in the “middle” of the network by being close to many friends.
- What metric could identify these central nodes?
  - Graph theory:

    Cen(G) = {v : e(v) is the smallest of all vertices in G}
  - Complex networks:

    Closeness centrality



https://www.geeksforgeeks.org/closeness-centrality-centrality-measure/

Closeness centrality: definition

Closeness centrality for node i is the average distance between a vertex *i* and all vertices in the graph (consider vertices in the same component only):

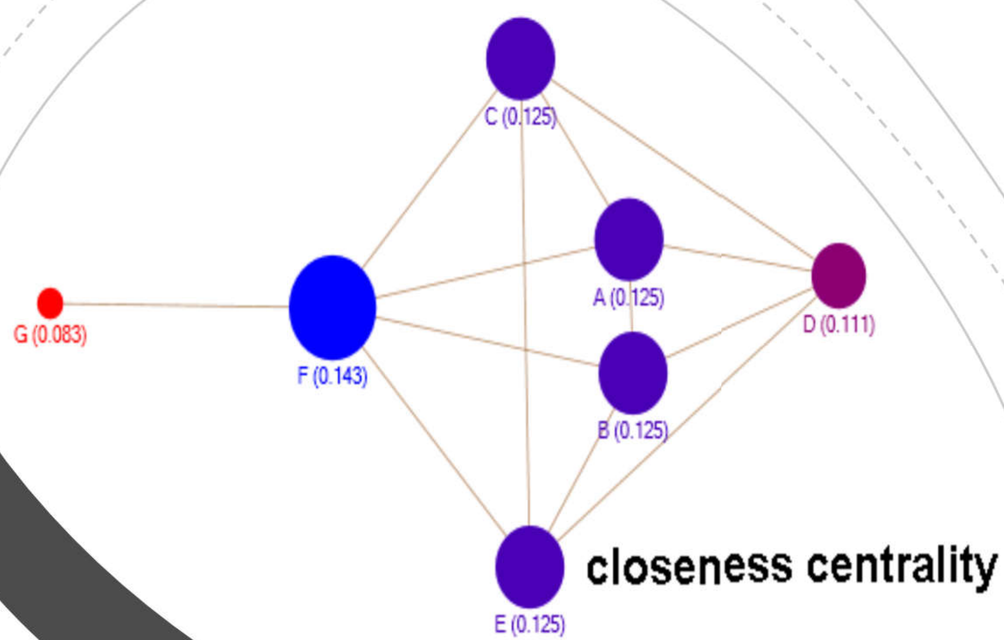
$$C_i = 1 \Big/ \sum_{j=1}^{n} d(i,j)$$

The formula depends on inverse distance to other vertices.

Closeness centrality can be viewed as the efficiency of a vertex in spreading information to all other vertices.

Drawback: only computed per component

Examples


C (0.125)
A (0.125)
G (0.083)
F (0.143)
D (0.111)
B (0.125)
E (0.125)
closeness centrality

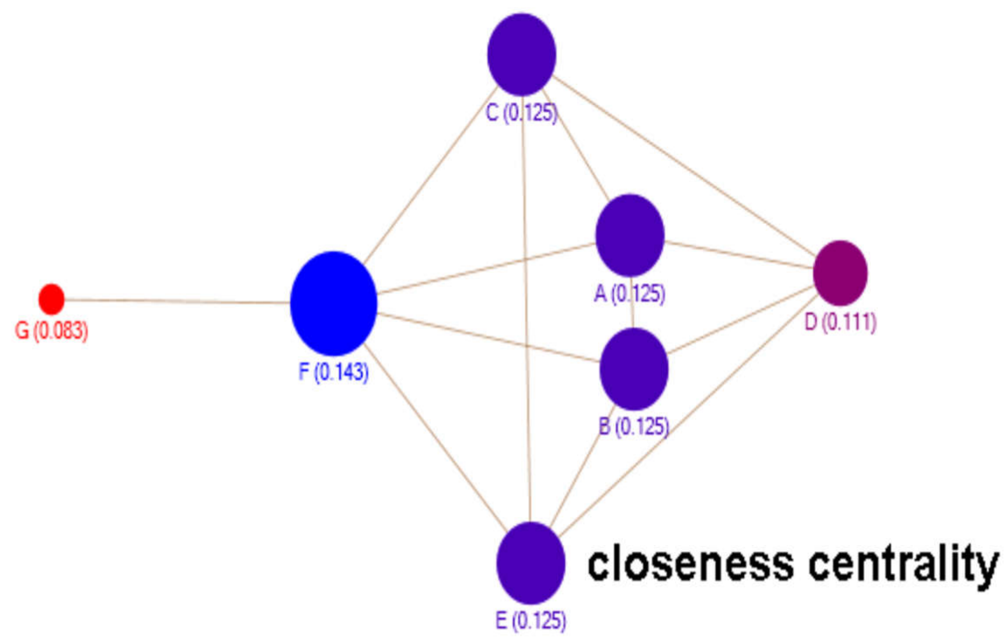
Closeness Centrality

$$C_i = 1 \Big/ \sum_{j=1}^{n} d(i,j)$$

$$C_A = \frac{1}{d(AB) + d(AC) + d(AD) + d(AE) + d(AF) + d(AG)}$$

$$C_A = \frac{1}{1+1+1+2+1+2}$$

$$C_A = \frac{1}{8} = 0.125$$



In class exercise: closeness centrality

- What is the centrality of a vertex in K_4?
- What is the centrality of a vertex in K_{14}?
- What is the centrality of a vertex in K_n?
- Should they be the same regardless of the *n*?

Sometimes, we care for a relative centrality, so it should be the same for all n values, since it identifies a certain structure.

- How would you fix the "problem" so that it scales with *n*?

$$C^{normalized}{}_i = \frac{n}{\sum_{j=1}^{n} d(i,j)}$$

where *n* is number of vertices in the graph.

- In class exercise: What is the normalized closeness centrality of a vertex in K_n?

$$C_i = 1 \Big/ \sum_{j=1}^{n} d(i,j)$$

Well Defined?!

Closeness centrality

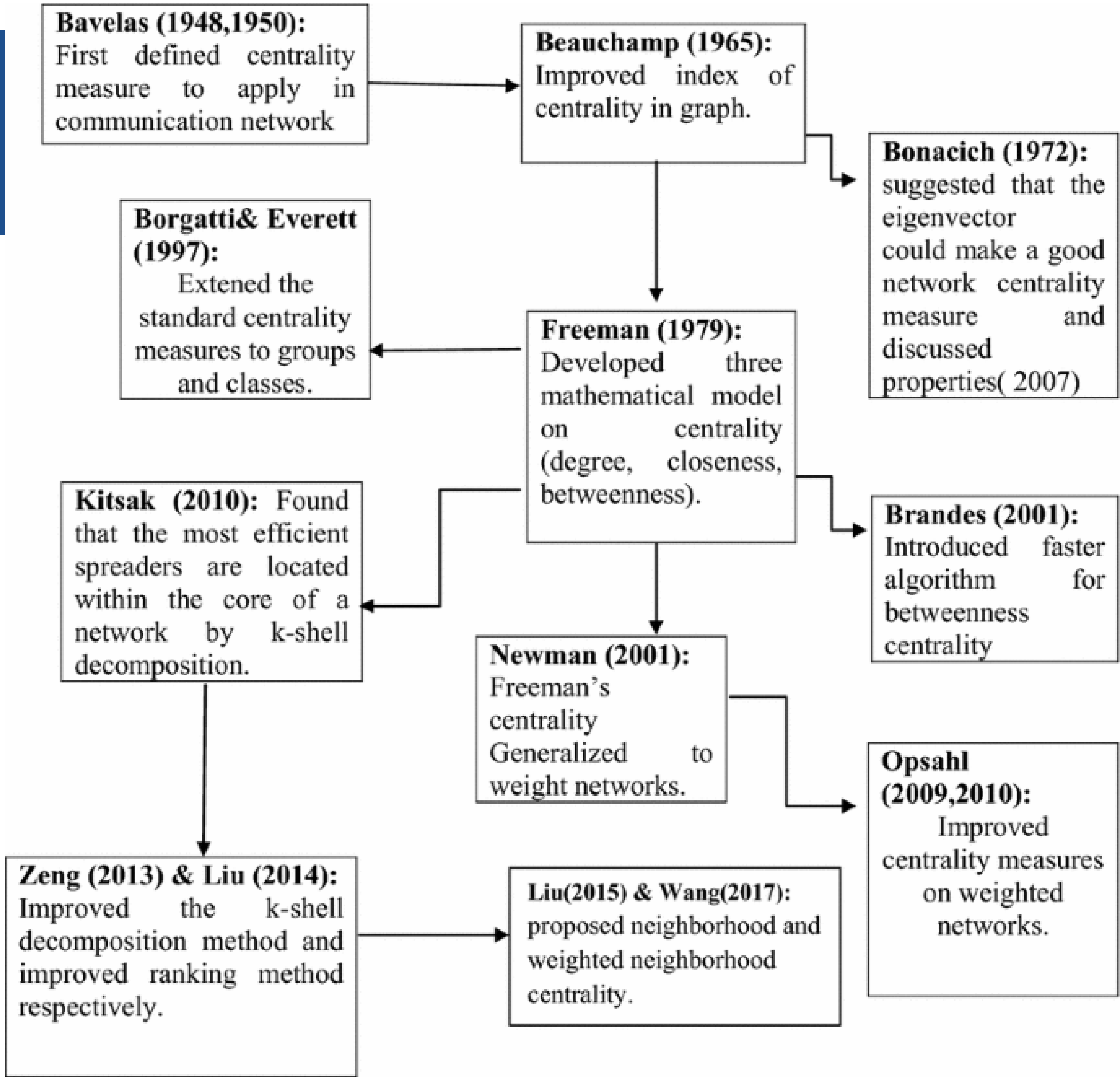
- In a typical network the closeness centrality might span a factor of five or less
  - It is difficult to distinguish between central and less central vertices
  - a small change in network might considerably affect the centrality order
- It is well defined?! Consider it in a disconnected network: $C_i = 1 \Big/ \sum_{j=1}^{n} d(i,j)$
- Alternative computations exist but they have their own problems:
  - Such as the harmonic mean: $C'_i = \frac{1}{n-1} \sum_j \frac{1}{d(i,j)}$

  Which works for disconnected graphs since $\frac{1}{d(i,j)}$ → 0 if *i* and *j* are in different components.

  But still small range of values for most networks.
- Both closeness centrality and harmonic closeness centrality are hardly ever used

Extensions

- How would you generalize the closeness centrality?
- This is a good time to share your thoughts



https://link.springer.com/article/10.1007/s13278-018-0493-2