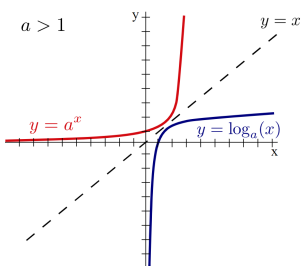
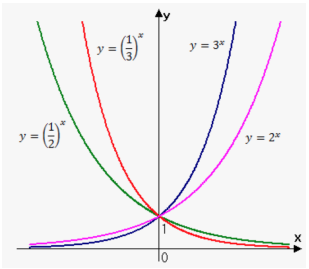
Appendix 2: Exponential and Logarithmic Functions

1. **Exponential Functions**: $f(x) = a^x$ where $a \in \mathbb{R}^+$ is the general form.
2. examples of exponential functions: $f(x) = 2^x$ or $g(x) = 2^{-x} = (\frac{1}{2})^x$
3. laws:
   - $a^{-x} = \frac{1}{a^x} = (\frac{1}{a})^x$
   - $b^{x+y} = b^x b^y$
   - $(b^x)^y = b^{xy}$
4. **Logarithmic Functions**: $f(x) = \log_a x$, where $a > 1$ (logarithm of x base a).
5. common bases: $2, 10, e$, for this book $\log x = \log_2 x$ (other books will imply $\log x = \log_{10} x$). Standard notation: $\ln x = \log_e x$
6. laws:
   (a) $\log_a a = 1$
   (b) $\log_a 1 = 0$
   (c) $a^{\log_a x} = x$
   (d) $\log_a(a^x) = x \log_a a = x$
   (e) $\log_a(xy) = \log_a x + \log_a y$ for $a > 1$, $x, y \in \mathbb{R}^+$
   (f) $\log_a(x^y) = y \log_a x$ for $a > 1$, $x \in \mathbb{R}^+$

   (g) $\log_a x = \dfrac{\log_b x}{\log_b a}$ for $a > 1, b > 1$, $x \in \mathbb{R}^+$

7. Graphs of the exponential and logarithmic functions:





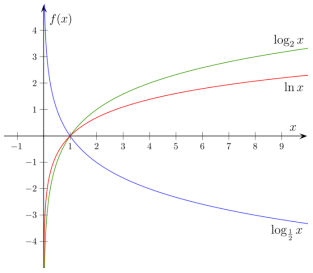


Figure 1: Examples of (a) General plots of log versus exponential functions for $a > 1$, (b) exponential functions for $a > 1$ and $a < 1$, and (c) logarithmic functions for $a > 1$ and $a < 1$