CH 1: Functions as models

1.1 Four ways to represent a function

This section introduces the notion of a function.

1. recall that $|a| = \begin{cases} a & a \geq 0 \\ -a & a < 0 \end{cases}$
2. a function is a rule that assigns to each element x of a set D a unique element, namely $f(x)$ in some set E.
3. the set D is called domain, and the set E is called the range (it is possible for D = E)
4. the variable x is the independent variable, and the value f(x) is the dependent variable
5. a function can be given or visualized as:
   (a) verbally by a description of words
   (b) numerically by a table of values for the independent and dependent variables
   (c) a visually by a graph given by the ordered pairs $\{(x, f(x)) : x \in D\}$ or an arrow diagram connecting the independent variables x to their images f(x)
   (d) a algebraically by an explicit formula together with a domain and codomain
6. in determining the domain of a function, one should pay attention to:
   (a) square roots: the quantity under the square root has to be ≥ 0
   (b) fractions: the denominator must be 0 (including $sinx$ and some other trig functions)
   (c) logarithms: the quantity that you must take the logarithm of has to be > 0
   (d) other particular restrictions might apply but the above are the most common ones
7. in determining the range of a function: consider the lower and the upper values the function can take as you test different values of input
8. testing if a formula or graph represents a function: Vertical line test: any vertical line should intersect the graph at most one time.
9. a function is called increasing on an interval [a,b] if $x < y \rightarrow f(x) < f(y)$, for all $x, y \in [a, b]$
10. a function is called decreasing on an interval [a,b] if $x < y \rightarrow f(x) > f(y)$, for all $x, y \in [a, b]$
11. a function is called constant on an interval [a,b] if f(x) = f(y), for all $x, y \in [a, b]$
12. some functions can be increasing on part of their domain, and decreasing on other parts, and constant on other parts