CH 1: Functions as models

1.5 Inverse Functions and Logarithms

This section presents the inverse functions.

1. a function is one-to-one if it passes the horizontal line test: no horizontal line intersects the graph more than once (i.e. does $f(a) = f(b)$ imply $a = b$).
2. if a function $f(x) = y$ is one-to-one with domain A and range B, then the inverse function on B exists and its formula is given by $f^{-1}(y) = x$. NOTE: $f^{-1}(x) \neq \frac{1}{f(x)}$, but rather it is the function that will undo whatever f did.
3. the graph of f^{-1} is obtained from the graph of f by reflecting the graph of f with the line $y = x$
4. properties of logs: $log_a x = y \iff a^y = x$, for all positive constants a
   (a) $log_a(xy) = \log_a x + \log_a y$
   (b) $log_a(\frac{x}{y}) = \log_a x - \log_a y$
   (c) $\log_a x^r = log_a(x^r) = \log_a(x)^r = r \cdot log_a x$
   (d) $\ln x = y \iff e^y = x$
   (e) $\ln e = 1$
   (f) $\ln e^x = x$ and generally $\log_a a^x = x$
   (g) $e^{\ln x} = x$ and generally $a^{\log_a x} = x$
   (h) change of base formula: $\log_a x = \frac{lnx}{\ln a}$
   (i) general change of base formula: $\log_a x = \frac{\log_b x}{\log_b a}$