

# 1 The Foundations: Logic and Proofs

## 1.1 Propositional Logic

1. a proposition is a declarative sentence that is either true (T) or false (F), but not both.
2. a sentence with a variable can be turned into a proposition if a value is assigned to the variable
3. a propositional variable is a variable that can be turned into a proposition by assigning value to it (Ex: propositional variables:  $P(x) : x > 0$ . The proposition  $P(-3) : -3 > 0$  is false)
4. propositional calculus or propositional logic is the area of logic that deals with propositions
5. compound propositions are new propositions obtained from already existing ones, using logical operators (or connectives: NOT, AND, OR, XOR, IF...THEN, IFF)
6. in deciding whether a sentence that involves a variable is T or F, we use fixed time, place and particular people
7. truth tables are tables that display all the possible choices for the propositions. For example, if only one proposition is involved, there are only two choices: T or F. If two propositions are involved, there are four choices to consider: both T, both F, first T and second one F, or first F and second one T (see tables page 4)
8. truth tables for the negation of p ( $\neg p$ ), p and q ( $p \wedge q$ ), p or q ( $p \vee q$ ), p exclusive or q ( $p \oplus q$ ), implication ( $p \rightarrow q$ ), biconditional ( $p \leftrightarrow q$ )
9. in an implication ( $p \rightarrow q$ ), p is the hypothesis and q is the conclusion
10. the converse of ( $p \rightarrow q$ ) is  $q \rightarrow p$
11. the inverse of ( $p \rightarrow q$ ) is  $\neg p \rightarrow \neg q$
12. the contrapositive of ( $p \rightarrow q$ ) is  $\neg q \rightarrow \neg p$
13. precedence of logical operators:  $\neg$ ; then  $\wedge$  and  $\vee$ , and then  $\rightarrow$  or  $\leftarrow$  or  $\leftrightarrow$
14. a boolean variable is a variable that is either T or F, so it could be regarded as a "1 or 0 type" of variable
15. a bit string is a sequence of zero or more bits
16. the length of a bit string is the number of bits in the string
17. OR, AND, and XOR can be done bit wise instead of using T and F