10 Graphs

10.3 Representing Graphs and Graph Isomorphism

1. Representing graphs:

   - using a drawing along with a description or the graph class
   - using adjacency list: list all the vertices, and then the edges that are incident with each vertex
   - using adjacency matrix: a matrix obtained by listing all the vertices along the top and the side of the matrix, and an entry a_{ij} of the matrix is 1 if and only if the two vertices v_i and v_j are adjacent. Properties:
     - The matrix is unique up to the labeling used on the vertices
     - For a graph, the adjacency matrix is symmetric
     - For a digraph, the adjacency matrix does not have to be symmetric.
   - using incidence matrix: a matrix that shows what edges are incident to what vertices. It is an $n \times m$ matrix, where n is the number of vertices, and m is the number of edges/arcs

2. The main question of the section is: How do we know if what may look like two different graphs is actually the same graph, just presented with a different drawing or a formula. This uses the concept of isomorphism

3. Two graphs G and H are isomorphic, if there is a bijection f from $V(G)$ to $V(H)$ that preserves adjacencies (i.e. u and v are adjacent vertices in G if and only if $f(u)$ and $f(v)$ are adjacent in H) and non-adjacencies.
   This function f is called an isomorphism from G to H.

4. How do we know if two graphs are isomorphic?

   - isomorphic: find a bijection f
   - not isomorphic: check for different number of edges, different number of vertices, different degree sequence in the two graphs, or show that one graph has a property that the other doesn't (for example one graph might be bipartite and the other might not)

5. a graph invariant is a property that is preserved by an isomorphism, such as the number of vertices, number of edges, degree of a vertex, same number of vertices of a particular degree, same number of cycles, whether the graphs are connected or not, and so on