11 Trees

11.1 Introduction to Trees

- 1. A <u>tree</u> is a connected graph without cycles (or without simple circuits).
- 2. A <u>rooted tree</u> is a tree in which one vertex has been designated as the root and every edge is directed away from the root (undirected, yet understood that edges point away from the root capturing the parent-child relationship):
 - The parent of v is the unique vertex u such that there is a directed edge from u to v (for each vertex, its parent is unique).
 - When u is the parent of v, v is called a <u>child</u> of u.
 - Vertices with the same parent are called siblings.
 - The <u>ancestors</u> of a vertex is the root and the vertices in the path from the root (excluding the vertex itself)
 - The <u>descendants</u> of a vertex v are those vertices that have v as an ancestor
 - A vertex of a rooted tree is called a <u>leaf</u> if it has no children.
 - Vertices that have children are called <u>internal vertices</u>.
 - The <u>height</u> of a rooted tree is the maximum of the levels of vertices (i.e. the length of the longest path from the root to all vertices)
- 3. A m-ary tree is rooted tree whose internal vertex has no more than m children.
- 4. The tree is a <u>full *m*-ary tree</u> if every internal vertex has exactly *m* children (if m = 2, it is called a binary tree).