2 Sets, Functions, Sequences, and Sums

2.2 Set Operations

1. When we talk about subsets, we are concerned with subsets of a larger set, usually called universal set denoted by U.

   We can use Venn Diagrams to represent a set:

   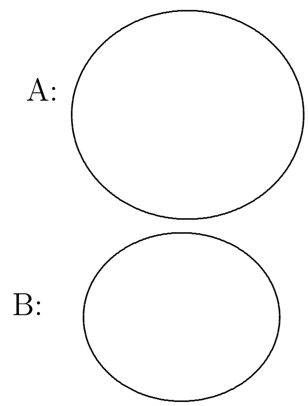

   Figure 1: A Venn Diagram of Disjoint Sets

   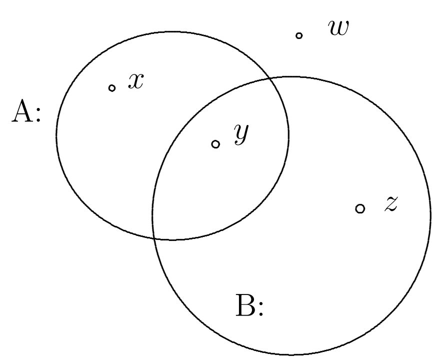

   Figure 2: A Venn Diagram of Intersecting Sets

   For Figure 2: $x \in A, y \in A, y \in B, z \in B, w \notin A, w \notin B$.

2. Let A and B be two sets. The following are ways of combining two or more sets:

   (a) The intersection of A and B: $A \cap B = \{a : a \in A \text{ and } a \in B\}$. If $A \cap B = \emptyset$, then A and B are disjoint.

   (b) The union of A and B: $A \cup B = \{a : a \in A \text{ or } a \in B\}$.

   (c) The difference of A and B (also relative complement of B with respect to A): $A \setminus B = \{a : a \in A \text{ and } a \notin B\}$.

   (d) The complement of A: $\bar{A} = \{a : a \notin A\} = U \setminus A$, where U is the universal set.

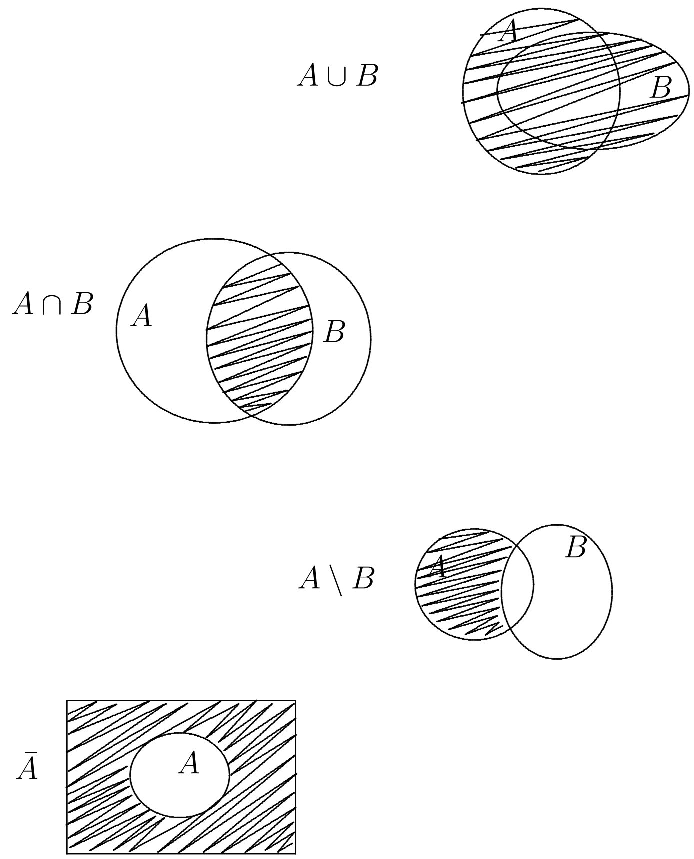

Figure 3: Set operations

Example: Let $A = \{1, 3, 5, 6, 7\}$, $B = \{1, 3, 8\}$, and the universal set $U = \{1, 2, \ldots, 10\}$. What are the intersection, union, difference...?

(a) $A \cap B = \{1, 3\}$.

(b) $A \cup B = \{1, 3, 5, 6, 7, 8\}$.

(c) the difference $A \setminus B = \{5, 6, 7\}$, and $B \setminus A = \{8\}$

(d) $\bar{A} = \{2, 4, 8, 9, 10\}$

3. set identities -page 130 (note that they are similar to the "or" and "and" tables for predicates)

4. $|A \cup B| = |A| + |B| - |A \cap B|$, which is the Inclusion Exclusion principle for two sets.

5. when proving inequalities, there are four choices of techniques:

   - "chasing the element" (see Example 10 page 130): In order to show that some set X is a subset of Y, we choose an arbitrary element $x \in X$, and we show that $x \in Y$ Borges, Carlos (CIV)(where X and Y could be expressions involving some sets, so for Example 10, $X = \overline{A \cap B}$, and $Y = \overline{A} \cup \overline{B}$)
   - "logical equivalences" (see Example 11 page 131): Use the definition to show the inequality in question

- "using the laws on page 130" (see Example 14 page 132): Use the laws to show the inequality in question
- "membership table" (See Example 13 page 130): This is like a truth table: you consider all the choices of A, B, and C, where x could be an element of each or not.

6. Generalized Intersection and Unions: Indexed Collection of Sets
Suppose that $A_1, A_2, \ldots, A_n$ is a collection of collection of sets, ($n \geq 3$). The following are ways of combining two or more sets:

   (a) The intersection of the n sets $A_1, A_2, \ldots, A_n$ is:

$$\bigcap_{i=1}^{n} A_i = \{x : x \in A_i, \forall i, 1 \leq i \leq n\}.$$

   (b) The union of of the n sets $A_1, A_2, \ldots, A_n$ is:

$$\bigcup_{i=1}^{n} A_i = \{x : x \in A_i, \exists i, 1 \leq i \leq n\}.$$

Example: Let $A_i = \{i, i+1\}$, $1 \leq i \leq 10$. What are the intersection and the union of them.

   (a) $\bigcap_{i=1}^{10} A_i = \emptyset$.

   (b) $\bigcup_{i=1}^{10} A_i = \{1, 2, \ldots, 11\}$.

Note: If we have different index sets, we have different results:
Let $A_i = \{i, i+1\}$, and the index set $I = \{1, 5, 10\}$. Then

   (a) $\bigcap_{i \in I} A_i = \emptyset$.

   (b) $\bigcup_{i \in I} A_i = \{1, 2, 5, 6, 10, 11\}$.