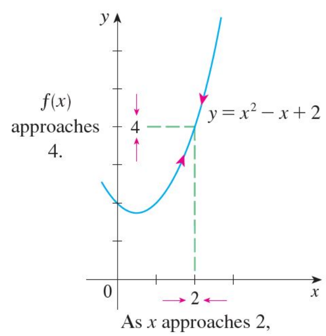
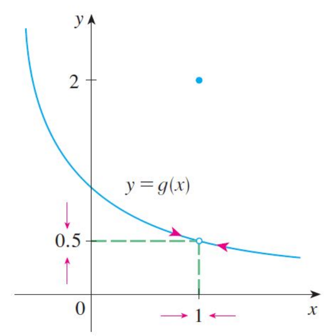
2.2 The limit of a function

1. Intuition: given a sequence of numbers, if they get closer to (potentially achieving) a value, we call that value the limit
2. For a function $f(x)$, the limit at the point $x = a$ is

   $\lim_{x\to a} f(x) = L$ if the function $f(x)$ approaches (or even attains) the value L as x approaches a.
3. when finding a limit of an expression, you should first plug in the value to see if it exists: example let $f(x) = x^2 - x + 2$, then $\lim_{x\to 2} x^2 - x + 2 = 4 = f(2)$.





4. the function may not be defined at the value or the value could be different than the limit:
   - **left limit**: $\lim_{x\to a^-} f(x)$ and
   - **right limit** $\lim_{x\to a^+} f(x)$.
   - if the left and the right limit are equal, then the function has a limit at that point: $\lim_{x\to 0^-} \frac{1}{|x|} = \infty$ and $\lim_{x\to 0^+} \frac{1}{|x|} = \infty$ and so $\lim_{x\to 0} \frac{1}{|x|} = \infty$.
   - if the left and the right limit are equal, then the limit does not exist (DNE) at that point: $\lim_{x\to 0^-} \frac{1}{x} = -\infty$ and $\lim_{x\to 0^+} \frac{1}{x} = \infty$, and so $\lim_{x\to 0} \frac{1}{x}$ DNE.
5. if the limit $\lim_{x\to a} f(x)$ is ∞ or $-\infty$ then the function has a vertical asymptote at $x = a$.
6. important limit: $\lim_{x\to 0} \frac{\sin x}{x} = 1$ (it will be proved in chapter 3)
7. As you plug in the value, you can get a nonzero number divided by 0, which will give you the limit to be ∞ or $-\infty$, and you need to use the left-hand and right-hand limit.
   Ex 1: $\lim_{x\to 0} \frac{1}{x^2} = \infty$ since $\lim_{x\to 0^+} \frac{1}{x^2} = \infty$ and $\lim_{x\to 0^-} \frac{1}{x^2} = \infty$.
   Ex 2: $\lim_{x\to 0} \frac{1}{x^3}$ DNE since $\lim_{x\to 0^-} \frac{1}{x^3} = -\infty$ and $\lim_{x\to 0^+} \frac{1}{x^3} = \infty$.
   Ex 3: $\lim_{x\to 0} \frac{-1}{x^2} = -\infty$ since $\lim_{x\to 0^+} \frac{-1}{x^2} = -\infty$ and $\lim_{x\to 0^-} \frac{-1}{x^2} = -\infty$.

8. Example: $f(x) = \frac{x+1}{x^2-1}$. Then $\lim_{x \to 0} f(x) = f(0) = -1$, however $\lim_{x \to -1} f(x) = -\dfrac{1}{2}$ (a calculator will give an incorrect value here!!!)

We next learn how to evaluate limits