

2.3 Calculating limits using the limit laws

This section introduces methods to evaluate limits

1. If the limits $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exists, then
 - (a) $\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$
 - (b) $\lim_{x \rightarrow a} [cf(x)] = c \lim_{x \rightarrow a} f(x)$
 - (c) $\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$
 - (d) $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$, where $\lim_{x \rightarrow a} g(x) \neq 0$
2. $\lim_{x \rightarrow a} (f^n(x)) = \lim_{x \rightarrow a} [(f(x))]^n = [\lim_{x \rightarrow a} (f(x))]^n$
3. $\lim_{x \rightarrow a} \text{constant} = \text{constant}$, and also $\lim_{x \rightarrow a} x = a$
4. $\lim_{x \rightarrow a} x^n = a^n$, if $n > 0$
5. $\lim_{x \rightarrow a} x^{\frac{p}{q}} = a^{\frac{p}{q}}$ as long as the p^{th} -root is defined
6. $\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)}$, for all functions $f(x)$ for which the n^{th} -root is defined.
7. if two functions are the same except at the point a where the limit is evaluated (i.e $f(x) = g(x)$ when $x \neq a$), then $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x)$
8. if $f(x) \leq g(x)$, then $\lim_{x \rightarrow a} f(x) \leq \lim_{x \rightarrow a} g(x)$
9. if $f(x) \geq g(x)$, then $\lim_{x \rightarrow a} f(x) \geq \lim_{x \rightarrow a} g(x)$
10. Squeeze Theorem: if $f(x) \leq g(x) \leq h(x)$, then $\lim_{x \rightarrow a} f(x) \leq \lim_{x \rightarrow a} g(x) \leq \lim_{x \rightarrow a} h(x)$.
 Particularly, if $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = L$ for some constant L , then we also obtain $\lim_{x \rightarrow a} g(x) = L$.

