2.5 Continuity

1. a function f is continuous at a number a if $\lim_{x\to a} f(x) = f(a)$ (i.e the function $f(x)$ approaches the value $f(a)$ as x gets closer to a). This means $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$

2. otherwise, a function could be continuous only on one side:

   (a) a function f is continuous at a number a from the left if $\lim_{x\to a^-} f(x) = f(a)$.

   (b) a function f is continuous at a number a from the right if $\lim_{x\to a^+} f(x) = f(a)$.

3. a function is continuous on an interval or set if it is continuous on every point in that interval

4. let f and g be continuous at a, then for some constant c these are continuous at a:

   (a) $f \pm g$

   (b) $f \cdot g$

   (c) $\frac{f}{g}$, if $g \neq 0$

   (d) constant multiple of either functions: cf and cg

   (e) composition of functions: $f \circ g$ and $g \circ f$

5. there are some classes of functions that are continuous everywhere on their **domains**:

   (a) polynomials / rational / root functions

   (b) exponential / logarithmic functions

   (c) $|x|$

   (d) $\sin x$ and $\cos x$ (but not $\tan x$)

   (e) ANY COMBINATION OF THE ONES ABOVE IS CONTINUOUS ON ITS DOMAIN. That is, you only need to check continuity at the points where the function has a zero denominator or more than one piece of a function combined

6. the **removable discontinuity** is one where $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$, but different than $f(a)$

7. the **jump discontinuity** is one where $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$

8. the **infinite discontinuity** is one where the function goes to infinity as x approaches a fixed value: $\lim_{x\to a^-} f(x) = \pm\infty$ and $\lim_{x\to a^+} f(x) = \pm\infty$

9. *The Intermediate Value Theorem:* If f is **continuous** on (a, b), then for each $N \in [f(a), f(b)]$, there is $c \in (a, b)$ such that $f(c) = N$. This means that a function will take all the image values between the numbers $f(a)$ and $f(b)$.

   - helps identify if functions have roots: if say $f(a) < 0$ and $f(b) > 0$, the function will have a root in the interval (a, b)
   - the value c may not be unique, and it guarantees the existence of the value c, it doesn't tell you how to find it.