2.7 Derivatives and Rates of Change

1. a first step in finding what the behaviour of the function is, is to look at the derivative at one point, say at some point that we call $(a, f(a))$, such as $(3, f(3))$ (or just at $x = 3$).

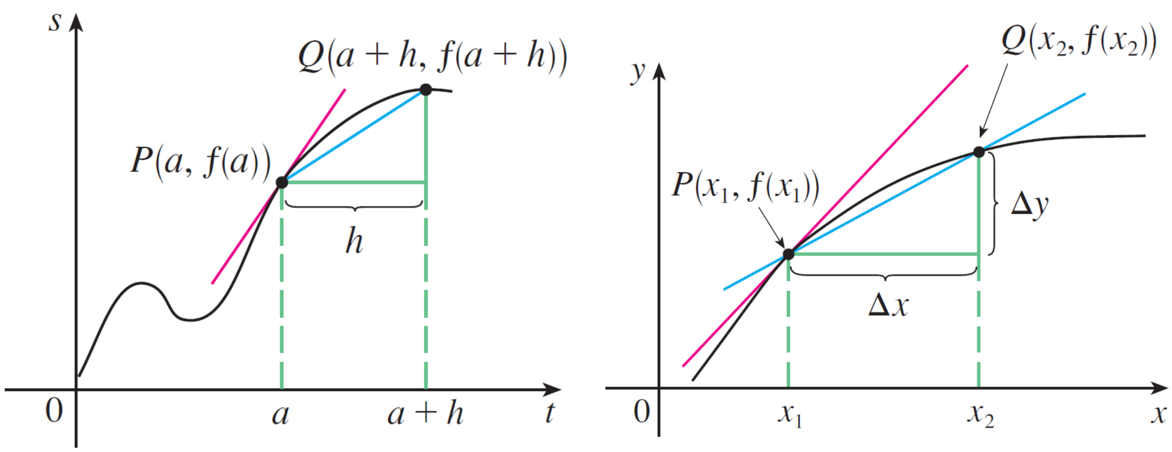
2. the slope of the tangent line to a curve at some point a gives the derivative of the function at the point a (for distance, the derivative gives the instantaneous rate of change, i.e. the speed). More exactly: if $f(x)$ is the curve, then the slope of the tangent line (if this limit exists) at the point $(a, f(a))$ is

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

3. another way of writing that is in terms of a little change h in the x-value:

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

   This formula can be obtained if $h = x - a$ in the one above it.



4. And so, we define the derivative of $f(x)$ at the point a to be exactly the above limit, if the limit exists:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h},$$

   and it gives the instantaneous rate of change with respect to x of the function $f(x)$ at the point a

5. now, one would have to apply this formula over and over for each value of a, but instead, Section §2.8 shows how to find the derivative at each point x, rather than just at one value a