2.8 The derivative as a function

1. we now define the derivative at every single point versus just one single point, namely the derivative function (it provides the rate of change at each point of the domain):
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

2. the derivative function is not always defined everywhere. For example if $f(x) = |x|$, then the derivative function does not exist at $x = 0$

3. for the derivative to exist at some point, the function must be continuous and **differentiable**, i.e. no corners like the absolute value, it cannot be discountinous, neither can it have a vertical tangent line (in all of these cases there is no slope of the tangent line at that point).

4. $f(x)$ is differentiable if its derivative $f'(x)$ exists, i.e. if the limit in the above definition exists.

5. and so, f must be continuous in order for f' to exist, but it is not sufficient. That is:
   - f differentiable $\Rightarrow$ f continuous
   - f continuous $\not\Rightarrow$ f differentiable (the counterexample is $f(x) = |x|$, which is one of the common counterexamples to differentiability.)

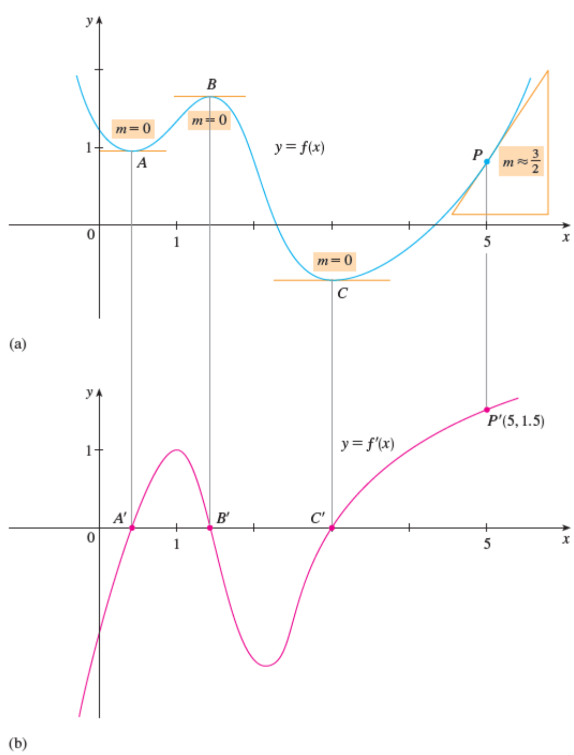

Figure 1: A function in blue, and its derivative in pink

6. the second derivative $f''(x)$ is the derivative of $f'(x)$. Similarly the 3rd derivative can be defined, and even higher orders. For distance, the second derivative is the acceleration, which shows how the speed changes instantaneously.