

3.2 The product and the quotient rules

1. first note that $\frac{d}{dx}(f(x) \cdot g(x)) \neq \frac{d}{dx}f(x) \cdot \frac{d}{dx}g(x)$ (i.e. $(f \cdot g)' \neq f' \cdot g'$).
Test this on $f(x) = x^3$ and $g(x) = x^{10}$

2. the **product rule**: $\frac{d}{dx}(f(x) \cdot g(x)) = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$, i.e.

$$(f \cdot g)' = f \cdot g' + f' \cdot g$$

3. similarly note that $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) \neq \frac{\frac{d}{dx}f(x)}{\frac{d}{dx}g(x)}$ (i.e. $(\frac{f}{g})' \neq \frac{f'}{g'}$). Check this on $f(x) = x^3$ and $g(x) = x^{10}$

4. the **quotient rule**: $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{g^2(x)}$, i.e.

$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$$