3.5 Implicit Differentiation

1. implicit differentiation works particularly well when the derivative of y must be found, but we cannot solve for y in terms of x in order to find y' (see example below)

2. used in finding the derivatives of functions of more than one variables, where the variables depend on each other. That is y is a function of x and chain rule must be used when taking the derivative of y with respect to x.

3. for example, if $x^3 + y^3 = 6xy$, find $\dfrac{dy}{dx}$ or y' by taking the derivative of both sides of the equation:

$$x^3 + y^3 = 6xy \tag{1}$$
$$3x^2 + 3y^2\frac{dy}{dx} = 6y + 6x\frac{dy}{dx} \tag{2}$$
$$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x} \tag{3}$$
$$\frac{dy}{dx} = \frac{2y - x^2}{y^2 - 2x} \tag{4}$$

4. the above solution can also be written as:

$$x^3 + y^3 = 6xy \tag{5}$$
$$3x^2 + 3y^2y' = 6y + 6xy' \tag{6}$$
$$y' = \frac{6y - 3x^2}{3y^2 - 6x} \tag{7}$$
$$y' = \frac{2y - x^2}{y^2 - 2x} \tag{8}$$