3.6 Derivatives of Logarithmic Functions

1. $(\log_a x)' = \frac{1}{x \ln a}$ (for $x > 0$)
2. $(\ln x)' = \frac{1}{x}$ (for $x > 0$)
3. $(\ln f(x))' = \frac{f'(x)}{f(x)}$ since $f(x)$ acts like x in the above equation, for $f(x) > 0$
4. $(\ln |x|)' = \frac{1}{x}$ (for $x \in \mathbb{R} - \{0\}$)
5. Logarithmic Differentiation: it is another method that helps find the derivative by first taking the natural log of both sides, and then taking the derivative of the new equation. This method works particularly well when we would have to use the quotient and product rule of very complicated expressions.
6. recall that e was that base of the log for which the derivative at $x = 0$ was 1. Here is another way to express it:
$$e = \lim_{x \to 0} (1 + x)^{1/x}.$$
Here is why: Let $f(x) = \ln x$. Then $f'(1) = 1$ or also
$$1 = f'(1) = \lim_{h \to 0} \frac{f(h+1) - f(1)}{h} = \lim_{h \to 0} \frac{\ln(h+1) - \ln 1}{h} = \lim_{h \to 0} \frac{1}{h} \cdot \ln(h+1) = \lim_{h \to 0} \ln(h+1)^{1/h}.$$
Taking e to the both sides we have the above relation.