4.4 L'Hospital Rule

When evaluating limits, plug in the value that x approaches. If this will produce an indeterminate, then use cancellation laws (section 2.3) or if the limit is at infinity divide by the highest power of x in the denominator (section 2.6). If neither works, then:

1. if $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}$ or $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{-\infty}{-\infty}$,

   then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$, i.e. take derivative of the numerator and denominator (not the quotient rule)

2. if $\lim_{x\to a} f(x)g(x) = 0 \cdot \infty$, then $\lim_{x\to a} f(x)g(x) = \lim_{x\to a} \frac{f(x)}{1/g(x)}$, i.e. write $g(x) = \frac{1}{1/g(x)}$.

3. if $\lim_{x\to a} f(x) - g(x) = \infty - \infty$, then use common denominator, or rationalization, or factorization to end up with an indeterminate of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

4. if $\lim_{x\to a} f(x)^{g(x)} = 0^0$, or $\lim_{x\to a} f(x)^{g(x)} = \infty^0$, or $\lim_{x\to a} f(x)^{g(x)} = 1^\infty$

(1) take ln of both sides (logarithmic differentiation): let

$$f(x)^{g(x)} = y$$

$$ln f(x)^{g(x)} = ln\ y$$
$$g(x) \cdot ln f(x) = ln\ y,$$

$$\lim_{x\to a} f(x)^{g(x)} = \lim_{x\to a} e^{ln\ y}.$$

(2) write the function as an exponential:

$$f(x)^{g(x)} = e^{ln f(x)^{g(x)}} = e^{g(x)\cdot ln f(x)}$$

$$\lim_{x\to a} f(x)^{g(x)} = e^{\lim_{x\to a} g(x)\cdot ln f(x)}$$