4.8 Newton's Method

1. It is well known that there are algebraic formulas that give the solutions of a linear, quadratic, cubic or quartic. However, there are no algebraic formulas that give solutions to polynomials of degree higher than 5 (abstract algebra classes will prove this) and similarly no algebraic formulas for transcendental equations (i.e. equations that involve other functions besides polynomials, like trig or exponentials)

2. Newton's method approximates roots of such equations using linear approximations.

3. say that you know that $f(x) = 0$ has a root, call it r (for example one way to know the root exists is by Intermediate Value Theorem, or the root of the derivative by Rolle's Theorem). Newton's method will describe how to find it:

   (a) we start with a guess, say x_1 of the root r

   (b) find the tangent line L at $(x_1, f(x_1))$

   (c) find the x-intercept of L and call it x_2 (x_2 is a better approximation of r than x_1 was)

   (d) repeat the steps above and obtain $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$, if $f'(x_n) \neq 0$

   (e) since larger n is, closer x_n is to r, we find $r = \lim_{n \to \infty} x_n$

You can watch an animation of Newton's method by clicking on this demo