5 Induction and Recursion

5.1 Mathematical induction (weak induction)

This section presents another proof technique (besides direct, contrapositive and contradiction)

1. Mathematical Induction is used to prove statements that are true for all positive integers or for positive integers greater than some number fixed positive integer.

2. The proof of a general statement $P(n), n \geq 1$ has two steps:

   (a) **Basis Step**: show that $P(1)$ is true The basis step is the foundation of the proof.

   (b) **Inductive Step**: $\forall k \geq 1 \left(P(k) \to P(k+1)\right)$ This step proves that every statement $P(k+1)$ is true given that the previous one, $P(k)$, is true. This implication is proved using a direct proof ($P(k)$ the the hypothesis).

3. So here is what the induction does: $P(1)$ is T as we showed in the basis step. Using the inductive step we obtain that $P(2)$ is T based on $P(k) \to P(k+1)$ when $k = 1$. And then, since we have that $P(2)$ is T, we use the inductive step again to get that $P(3)$ is T based on $P(k) \to P(k+1)$ when $k = 2$. And continuing in this manner, one may see how $P(n)$ is T by building up on the basis step and proving that we can go from any $P(k)$ to $P(k+1)$, $k \geq 1$.

4. What is the difference between n and k?

5. The basis step may not always be exactly $P(1)$. For example, if the results says to show that a statement is true for the positive integers greater than or equal to some number n_0, then $P(n_0)$ is what we have to show in the basis step

6. Types of problems that we will prove by mathematical induction:

   - summations
   - inequalities
   - divisibility