CH 5: Integrals

5.1 Areas and distances

1. area under a curve (i.e. between the graph of a function and the x-axis, generally the function is considered above the x-axis so the area is below the curve) can be found by dividing the region into rectangles. If the rectangles are all below the curve, then we underestimate the area. If they are above the curve, then we overestimate the area

2. smaller the width of the rectangles, better the estimation is. That is, if the width of the interval we want to computer the area of is divided into n rectangles, then as $n \to \infty$ we have exactly the area under the curve:

$$\lim_{n\to\infty} (\text{sum of the area of the approximating rectangles}) = \text{area under the curve.}$$

3. let L_n be the rectangles whose height is found with the left endpoints of each rectangle lying on the curve $f(x)$. Similarly, let R_n be the rectangles whose height is found with the right endpoints of each rectangle lying on the curve $f(x)$, and let M_n be the rectangles whose height is found with the midpoints of each rectangle lying on the curve $f(x)$. then

$$\lim_{n\to\infty} L_n = \lim_{n\to\infty} R_n = \lim_{n\to\infty} M_n = \text{area under the curve.}$$

4. let $\Delta x = \frac{b-a}{n}$, i.e. the constant width of each rectangle. Then the points that give the heights in an L_n approximation are:

$$a, x_1 = a + \Delta x, x_2 = a + 2\Delta x, \ldots$$

5. the area under the curve $f(x)$ is

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \lim_{n\to\infty} L_n = \lim_{n\to\infty} M_n,$$

usually we use the $\lim_{n\to\infty} R_n$ but all limits are equal.