5 Induction and Recursion

5.2 Strong Induction

This section presents another proof technique, which is more powerful than standard math induction.

1. Strong Induction is used to prove statements that are true for all positive integers or for positive integers greater than some number fixed positive integer.

2. The proof of a general statement $P(n), n \geq 1$ has two steps:

   (a) **Basis Step**: show that $P(1)$ is true The basis step is the foundation of the proof.

   (b) **Inductive Step**: $\Big(P(1) \wedge P(2) \wedge P(3) \wedge \ldots \wedge P(k)\Big) \rightarrow P(k+1)$

   This step proves that the statement $P(k+1)$ is true given that all the previous ones are true, namely, $P(1) \wedge P(2) \wedge P(3) \wedge \ldots \wedge P(k)$. The hypothesis is $P(1) \wedge P(2) \wedge P(3) \wedge \ldots \wedge P(k)$ and the conclusion is $P(k+1)$.

3. So what is the difference?

   - We can use all the previous statements $P(1), P(2), \ldots P(k)$ to prove the next $P(k+1)$
   - Because we use more thank just $P(k)$, sometimes more than just $P(1)$ needs to be checked, namely we need to check the first j values in the basis step, if we use $P_k, P_{k-1}, \ldots, P_{k-j}$. For example, if we use $P(k)$ and $P(k-1)$ to prove $P(k+1)$, then the first two values need to be verified in the basis step, such as $P(1)$ and $P(2)$. Similarly, if we use $P(k), P(k-1)$ and $P(k-2)$ to prove $P(k+1)$, then the first three values need to be verified in the basis step, such as $P(1), P(2)$ and $P(3)$

4. The basis step may not always be exactly $P(1)$. For example, if the results says to show that a statement is true for the positive integers greater than or equal to some number n_0, then $P(n_0)$ is what we have to show in the basis step