5.2 The Definite Integral

1. The definite integral of a function $f(x)$ from $x = a$ to $x = b$ is

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

   where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$ provided that the limit above exists. If it does exits, then the function is **integrable**, and the value of the integral represents the area under the curve $f(x)$ on the interval $[a, b]$. If the limit doesn't exist, then the function is **not integrable**.

2. If the function is continuous on $[a, b]$ then it is integrable.

3. also, if a function has only finitely many jump discontinuities, then the function is integrable

4. in evaluating the above limits, one will make use of the following summations:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

5. the area that is below the x-axis counts as negative area

6. properties of integral:

   (a) $\int_a^b c\ dx = c(b-a)$, where c is a constant

   (b) $\int_a^b f(x) + g(x)\ dx = \int_a^b f(x)\ dx + \int_a^b g(x)\ dx$

   (c) $\int_a^b f(x) - g(x)\ dx = \int_a^b f(x)\ dx - \int_a^b g(x)\ dx$

   (d) $\int_a^b c \cdot f(x)\ dx = c \int_a^b f(x)\ dx$

   (e) if $f(x) \geq 0$ when $x \in [a, b]$, then $\int_a^b f(x)\ dx \geq 0$

   (f) if $f(x) \geq g(x)$ when $x \in [a, b]$, then $\int_a^b f(x)\ dx \geq \int_a^b g(x)\ dx$

   (g) if $\min f(x) \leq f(x) \leq \max f(x)$ when $x \in [a, b]$, then $(\min f(x)) \cdot (b-a) \leq \int_a^b f(x)\ dx \leq (\max f(x)) \cdot (b-a)$

   However, rules (b) and (c) above do not work for multiplication and division, just like they didn't work for derivatives. We will learn techniques for that soon.