5.4 Indefinite Integral and the Net Change Theorem

1. Recall the **definite integral** is $\int_a^b f(t)\,dt = F(b) - F(a)$, and the answer is a **number** that gives the area under the function $f(x)$ from point $x = a$ to $x = b$.

2. If F is the antiderivative of f, then **the indefinite integral** is $\int f(t)\,dt = F(t)$, and the answer is a **a function**.

$$\int cf(x)\,dx = c\int f(x)\,dx \qquad \int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int k\,dx = kx + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1) \qquad \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C \qquad \int b^x\,dx = \frac{b^x}{\ln b} + C$$

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C \qquad \int \csc x \cot x\,dx = -\csc x + C$$

$$\int \frac{1}{x^2+1}\,dx = \tan^{-1}x + C \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$$

$$\int \sinh x\,dx = \cosh x + C \qquad \int \cosh x\,dx = \sinh x + C$$

3. the net change theorem: $\int_a^b F'(t)\,dt = F(b) - F(a)$, where note the the integrand F' is the derivative of the right hand side function F. Again, area below the x-axis is negative, so it gets subtracted off the area above the x-axis.

4. if the function F' is a velocity for example, then F is displacement, and it can be negative or positive depending on which direction the object moves