5.5 The Substitution Rule

1. If $u = g(x)$ is a differentiable function, then the reverse of chain rule works: $\displaystyle\int f\Big((g(x))\Big)g'(x)\ dx = \int f(u)\ du$

2. Notice how this works with $u = 3x$ so $du = 3\ dx$

$$\int \sin(3x) \cdot 3\ dx = \int \sin u\ du = -\cos(u) + C = -\cos(3x) + C$$

3. if we only have

$$\int \sin(3x)\ dx = \frac{1}{3}\int \sin(3x) \cdot 3\ dx = \frac{1}{3}\int \sin u\ du = -\frac{1}{3}\cos(u) + C = -\frac{1}{3}\cos(3x) + C,$$

   where $u = 3x$ so $du = 3\ dx$

4. for definite integrals, when $u = 3x$ so $du = 3\ dx$, we also change the limits:
   when $x = 0$ then $u = 3 \cdot 0$, and when $x = \pi/2$ we have $u = 3 \cdot \pi/2$:

$$\int_0^{\pi/2} \sin(3x) \cdot 3\ dx = \int_{3\cdot 0}^{3\cdot\pi/2} \sin u\ du = -\cos(u) + C = -\cos(3x)\Big|_0^{3\pi/2} = -(0-1) = 1,$$

5. Two shortcuts for an integral over a symmetric interval $[-a, a]$:
   (a) if f is even (i.e. $f(-x) = f(x)$) then $\int_a^a f(x)\ dx = 2\int_0^a f(x)\ dx$, which might be easier to computer at 0 instead of $-a$
   (b) if f is odd (i.e. $f(-x) = f(x)$) then $\int_a^a f(x)\ dx = 0$. because it is symmetric with the origin, and the area below the x-axis is negative.