6 Counting

6.1 The Basics of Counting

1. The basic principles of counting shown below generalize to more than 2 sets.

2. The **product rule** counts the number of ways a choice can be made from each of two sets of alternatives: Suppose a procedure can be broken down into a sequence of two tasks, where A is the set of choices for task 1, and B is the set of choices for task 2. Then the total number of choices for performing the procedure is

$$|A \times B| = |A||B|$$

   Example: On a restaurant's menu there are 2 choices of soup and 4 choices of salads. Then the number of choices of soup **and** salad: $2 \cdot 4 = 8$.

3. The **sum rule** counts the number of ways a single choice can be made from two disjoint sets of alternatives: If

$$A \cap B = \emptyset \;\rightarrow\; |A \cup B| = |A| + |B|$$

   Example: On a restaurants menu there are 2 choices of soup and 4 choices of salads. Then the number of choices of soup **or** salad: $2 + 4 = 6$ choices (note that there is no overlap between the choices).

4. The **Principle of Inclusion-Exclusion** or the subtraction rule extends the sum rule to situations in which the two sets of alternatives are not disjoint. If two sets A and B are not disjoint, say

$$A \cap B \neq \emptyset \;\rightarrow\; |A \cup B| = |A| + |B| - |A \cap B|$$

   To see this, note that if $x \in A \cap B$, then x is counted twice in $|A| + |B|$, namely once in $|A|$ and once in $|B|$. So we must subtract $|A \cap B|$ to correct the overcount.

5. The **division rule**: There are

$$\frac{n}{d}$$

   ways to do a task if it can be done using a procedure that can be carried out in n ways, and for every way w, exactly d of the n ways correspond to way w.

   In other words, in a collection of n colored objects, where each color has the same number of d elements, there are $\frac{n}{d}$ different colors. The division rule allows us divide the total number by the number of duplications (it is the multiplication principle as a division).