6 Counting

6.2 The Pigeonhole Principle

1. The Pigeonhole Principle says that if $k+1$ or more objects are placed into k boxes, at least one box must contain at least two pigeons.

2. The Generalized Pigeonhole Principle If n objects are placed into k boxes, then there is at least one box that contains at least $\lceil n/k \rceil$ objects.

3. What is the smallest n such that at least one of k boxes must contain at least r of n objects? In order to have at least r objects into a box, we need

$$\lceil n/k \rceil \quad \geq \quad r$$

So the smallest n that forces a specific box to contain r of n objects is $n = k(r-1)+1$.

Examples:

(a) Show that in a group of eight people there must be two whose birthdays fall on the same day of the week.

(b) In a dresser drawer there are socks in two colors, say blue and gray. If you cannot see the color of the socks, how many socks must you grab to guarantee that you have a pair of the same color?

(c) In a dresser drawer there are socks in three colors, say blue, white and gray. If you cannot see the color of the socks, how many socks must you grab to guarantee that you have a pair of the same color?

(d) In a dresser drawer there are 6 blue socks, 6 white socks and 6 gray socks. If you cannot see the color of the socks, how many socks must you grab to guarantee that you have a pair of the white socks?