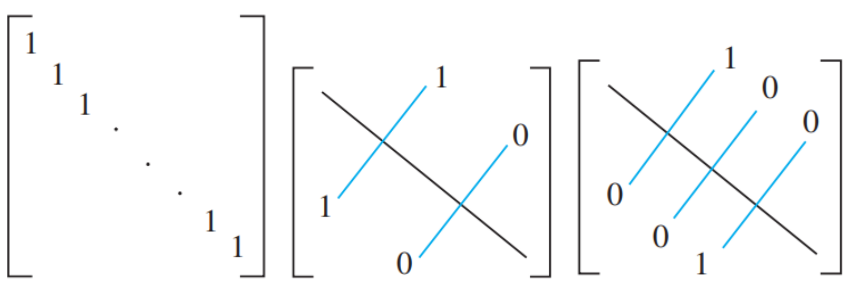
9 Relations

9.3 Representing Relations

Representing relations using matrices

1. A zero-one matrix $M_R = [m_{ij}]$ can be used to represent a relation $R = \{(a_i, b_j) \text{ for some } i, j\}$ if we let $m_{ij} = 1$ iff $(a_i, b_j) \in R$.
2. If R is a relation defined on a set, then M_R is a square matrix.
3. R is reflexive iff M_R has only 1s on its diagonal
4. R is symmetric iff $M_R = (M_R)^T$ (i.e. M_R is a symmetric matrix)
5. R is antisymmetric iff M_R has either $m_{i,j} = 0$ or $m_{j,i} = 0$ (or both) for all $i \neq j$



(a) reflexive (b) symmmetric (c) antisymmetric

6. R is transitive iff $1 \in M_R \odot M_R \; \rightarrow \; 1 \in M_R$
7. $M_{R\cup S} = M_R \vee M_S$ and $M_{R\cap S} = M_R \wedge M_S$ and $M_{R\circ S} = M_S \odot M_R$, $M_{R\oplus S} = M_R \oplus M_S$, and $M_{R^n} = M_R^{[n]}$

Representing relations using digraphs

1. A digraph consists of a set V of vertices (nodes) and a set E of ordered pairs (arcs): if $a, b \in V$ and $(a, b) \in E$ then the arc (a, b) (or just (ab)) belongs to the digraph, where a is the initial vertex and b is the terminal vertex of the arc
2. Each arc represents an element of the relation on A (The arc (a, a) is a loop)
3. R is reflexive iff $(a, a) \in E, \forall a \in A$
4. R is symmetric iff $(a, b) \in E \rightarrow (b, a) \in E$
   (if $(b, a) \in E$ and $(a, b) \in E$, then the two arcs collapse into an undirected edge)
5. R is antisymmetric iff $(a, b) \in E \;\; \oplus \;\; (b, a) \in E$, $\forall a, b \in A$
6. R is transitive iff $(a, b) \in E \;\; \wedge \;\; (b, c) \in E \;\; \rightarrow (a, c) \in E$
   (if $(a, b) \in E$ and $(b, a) \in E$ then you need $(a, a) \in E$ and $(b, b) \in E$)