9 Relations

9.5 Equivalence Relations

1. An equivalence relation R on A is a relation that is reflexive, symmetric and transitive

2. For an equivalence relation R, the equivalence class $[a]_R$ (or simply $[a]$) of an element $a \in A$ is the set containing a together with all elements related to a:

$$[a] = \{s : (a, s) \in R\} = \{s : (s, a) \in R\}$$

3. Any element of the class can be a representative of the class, and so any element of the class can give the name of the class (the name of an equivalence class is not unique)

4. In particular, the equivalence classes for the equivalence relation "congruence modulo m" are called congruence classes modulo m

5. A partition of a set A is a collection of subsets A_i $(1 \leq i \leq t)$ of A such that

   - any two subsets are disjoint (i.e. $A_i \cap A_j = \emptyset, \forall i \neq j, 1 \leq i \neq j \leq t$)
   - no subset is empty (i.e. $A_i \neq \emptyset, \forall i, 1 \leq i \leq t$)
   - the union of the subsets is A itself $\left(\text{i.e. } A = \bigcup_{i=1}^{t} A_i\right)$

6. The equivalence classes of an equivalence relation R on A form a partition of the set A

7. Similarly, a partition of A induces an equivalence relation R whose equivalence classes are the partition sets (and thus the relation can be obtained)