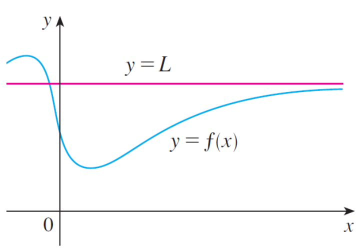
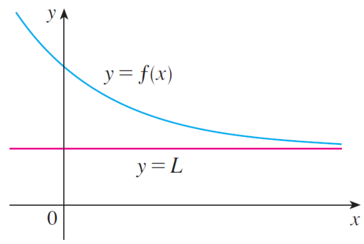
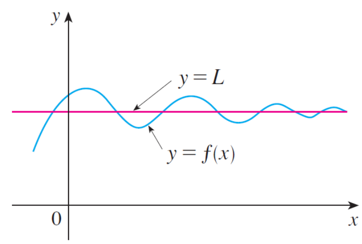
CH 2: Limits and Derivatives

2.6 Limits at Infinity: Horizontal Asymptotes

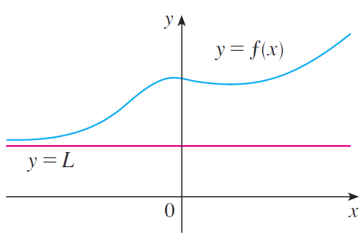
1. If f is defined on some interval (a, ∞), then $\lim_{x\to\infty} f(x) = L$ if $f(x)$ approaches L as x takes very large values.

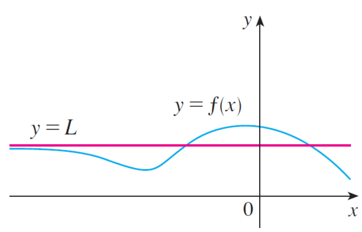


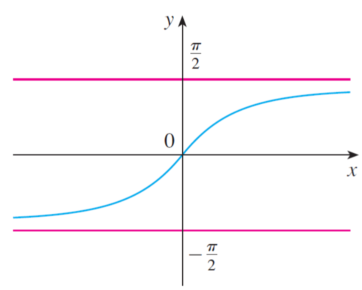




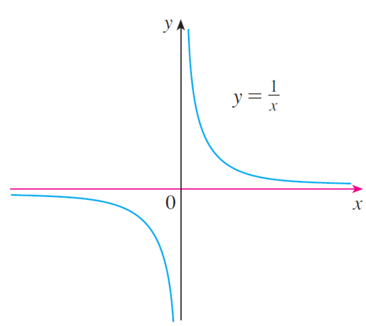
2. If f is defined on some interval $(-\infty, a)$, then $\lim_{x\to-\infty} f(x) = L$ if $f(x)$ approaches L as x takes very small values (or large negatives).







3. examples: $\lim_{x\to\infty} \dfrac{1}{x^n} = 0$ and $\lim_{x\to-\infty} \frac{1}{x^n} = 0$ You can see this from the graph of $\frac{1}{x}$



4. computing the limits at infinity: divide by the highest power of x in the denominator and use $\lim_{x\to\infty} \dfrac{1}{x^n} = 0$ if $n \geq 0$. The numerator must be any nonzero number. Similarly for $x \to -\infty$
5. the following are not defined: $\infty - \infty$, $0 \cdot \infty$, $\pm\frac{\infty}{\infty}$, $\frac{0}{0}$
6. these are defined: $\infty + \infty = \infty$ $\quad -\infty - \infty = -\infty$ $\quad e^{\infty} = \infty$ $\quad \infty \cdot \infty = \infty$ $\quad -\infty \cdot \infty = -\infty$