MA 4027: GRAPH THEORY AND APPLICATIONS

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Web-site : [http://faculty.nps.edu/rgera/MA4027.html](http://faculty.nps.edu/rgera/MA4027.html)
Learning Management System for the course: Microsoft Teams
Office Hours: Mon-Thu 11-11:50am (or drop in Microsoft Teams by texting me any time, and expect my reply within a couple of hours, at most 24 hours, including weekends and nights).

Course Description  Any successful complex system started as a simple system, that can often be modeled by a graph! This course focuses on the traditional graph theory knowledge and analysis of simple graphs that model simple systems, preceding the complex networks course studying complex networks that model complex systems. The graph theory course presents a selection of commonly used topics and proof techniques, providing the basis for understanding and performing analysis of patterned networks. This graph theory course provides a professional sandbox for the exploration of new terminology and results in discrete mathematics, with applications in many areas of the computing, social, and natural sciences.

The physical representation of elements (objects or live entities) and relationships between them, may provide more insight than just the data itself (think of a map of airline flights versus a table display of the same flights), so one can think of graph theory as a method for knowledge compression. Generally, graph theoretic representations of problems are useful when there is an emphasis on structure or structural properties of the graph modeling some data of interest. Algorithms and applications are included, but the focus is on understanding the structure of graphs and the techniques used to analyze the problems, thus solidifying the basics that learners can expand on in the future. The intellectual discipline of justifying an argument is valuable independently of mathematics, consequently it is important for learners to further develop this skill.

The course is designed to provide a comprehensive introduction to graph theory to students of operations research, electrical engineering, computer science, and mathematics. We studying what today is regarded as classical graph theory.

Course Intention  I designed the course to invite you, the student, to explore topics at your own pace, driven by your interests, while learning the essentials of graph theory. I structured it using both necessary common elements and choices on how you can participate and prove competency. The common element of the class is the weekly time spent together on practice problems, proofs, and data analysis. For the online learners, I make myself available weekly for you to meet with me as needed (details in Section 2).

This course empowers you to take control of your learning journey, allowing you the opportunity to select the activities that help you better understand concepts. Consequently, your focus transition from concerns about grades, to rather cultivating and expanding your mathematical knowledge. To support this flexibility, I provide choices in how you engage with the content, and how that assessment can take place through CHUNK Learning that I will further discuss in this document. Additionally, all content, homework and test allowing you to work at your own pace (of course, before the weekly deadlines).
1 Course Learning Outcomes and Objectives

Your goal for this class is to develop the mathematical sophistication needed to understand what properties to search for in graphs/networks, to perform network analysis, and to prove statements/claims about their graphs’ structure. To perform this, you must:

- Remember definitions, construct examples, and to distinguish examples from non-example.
- Apply concepts to validate intuition and independent mathematical thinking in problem solving.
- Reason from definitions to construct mathematical proofs by integrating core theoretical knowledge.
- Evaluate and synthesize published research papers.
- Analyze new networks using the main concepts of graph theory.
- Read and write graph theory in a coherent and technically accurate manner.

The learning outcomes above are achieved through proving results, analyzing data, reading scientific papers and creating new content to teach others. These learning objectives provide you the mathematical sophistication and confidence to use them in the future.

2 Course Format and Schedule: HyFlex and Student-centric

This course has two distinct features that might be new to you: HyFlex learning and student-centric teaching.

The HyFlex learning provides choices for the learners to engage with the content as follows: each student has the choice of participating in each class in one of the following three modalities.

<table>
<thead>
<tr>
<th>Student’s Choice</th>
<th>Face-to-Face</th>
<th>Synchronous Online via Microsoft Teams</th>
<th>Asynchronous Online via Microsoft Teams</th>
</tr>
</thead>
<tbody>
<tr>
<td>Learning options</td>
<td>A combination of CHUNK Learning micro-lectures and live instruction</td>
<td>A combination of CHUNK Learning micro-lectures and live instruction</td>
<td>CHUNK Learning micro-lectures and Review recordings of the live instruction.</td>
</tr>
<tr>
<td></td>
<td>Live instruction, and chat thread discussions in our course’s Teams site</td>
<td>Live instruction, and chat thread discussions in our course’s Teams site</td>
<td>Available weekly check-ins with me, and chat thread discussions in our course’s Teams site</td>
</tr>
<tr>
<td>Participation points</td>
<td>Class and chat thread discussions in our course’s Teams site</td>
<td>Class and chat thread discussions in our course’s Teams site</td>
<td>Chat thread discussions in our course’s Teams site</td>
</tr>
<tr>
<td>Homework Tests points</td>
<td>Practice problems</td>
<td>Take-home tests</td>
<td></td>
</tr>
<tr>
<td>Project points</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Office hours</td>
<td>F2F or online via Microsoft Teams</td>
<td>Online via Microsoft Teams</td>
<td>Online via Microsoft Teams</td>
</tr>
</tbody>
</table>

The student-centric teaching style uses an NPS created proof-of-concept platform called CHUNK Learning, which moves away from the static learning ecosystem towards a dynamic one that aims at personalized learning. Please explore the motivation and information on CHUNK Learning, and we will follow up with an introduction and discussion during the first week of classes.

The course is scheduled Mon-Thursday, time that we will use as follows:
• **Mondays’ and Tuesdays’ self guided learning**: All students will asynchronously explore and learn each week’s concepts using CHUNK Learning (also linked from the Microsoft Teams’ class site). Each week we cover a new chunk of content in CHUNK Learning, that we further discuss and explore on Wed and Thu through synchronous instruction. I am available for one-on-one sessions during Mondays’ and Tuesdays’ class time or for asynchronous conversations any time, just text me in Microsoft Teams.

• **Wednesdays’ problem based learning**: Synchronous discussions, problem solving and proof writing sessions to ensure understanding and theoretical practice of the explored concepts of Monday and Tuesday (examples, proofs, analysis). Recordings of these sessions will be available for support outside of the class. Asynchronous learners will also have access to these recording, and must also participate in the Microsoft Teams’ threaded discussions based on readings or classroom conversations.

• **Thursdays’ data exploration**: Synchronous meetings to analyze datasets using Gephi/Python/R. This analysis can be incorporated in the PPT slide for the [Network Profile Summary slide], an assessment choice discussed in Section 4. Recordings of these sessions will be available for support outside of the class. Asynchronous learners will also have access to these recording, and must also participate in the Microsoft Teams’ threaded discussions based on readings or classroom conversations.

Course Content: Course materials can be found in a network view directly at [www.CHUNKLearning.net](http://www.CHUNKLearning.net) or in a linear view at [http://faculty.nps.edu/rgera/MA4027.html](http://faculty.nps.edu/rgera/MA4027.html). The schedule for the course is the following:

<table>
<thead>
<tr>
<th>Week</th>
<th>Chapter</th>
<th>TOPIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>Overview, Software and STEM Reading</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>Introduction to Graph Theory</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>Degrees in Graphs</td>
</tr>
<tr>
<td>4</td>
<td>12, 3</td>
<td>Distance &amp; Isomorphic Graphs</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>Trees</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>Connectivity</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>Project: CHUNKlets creation</td>
</tr>
<tr>
<td>8</td>
<td>6, 7</td>
<td>Eulerian &amp; Hamiltonian;</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>Planarity</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>Coloring</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>CHUNKlets presentations</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>Individual discussions as needed. All CHUNKlets due on Wednesday</td>
</tr>
</tbody>
</table>

3 Homework

For Wednesday’s problem-based learning sessions listen to [www.CHUNKLearning.net](http://www.CHUNKLearning.net)’s videos for that week (complemented by the textbook) prior to working on homework to properly prepare for Wednesday’s class discussion of the homework problems. This allows the class time to be spent on practice problems, discussions, and group work. Homework [has already been posted](http://faculty.nps.edu/rgera/MA4027.html) and while none is collected it will be discussed on Wednesdays (or asynchronously in Teams’ chat) to provides checks on knowledge and your participation points.

For Thursdays’ data exploration sessions be prepared to use the following analysis software:
1) The primary tool is Gephi which is open source graph visualization and manipulation software.
2) NetworkX package in Python a Python library developed for the study of graphs and networks. Python is an open source programming language with wide interoperability and many, varied tools.
3) Other software choices are available and you are welcome to use them, such as igraph in R or Statnet with a tutorial here or here is a longer list.
4 Activities that support your learning

The class grade is a combination of the following three activities, detailed below:

<table>
<thead>
<tr>
<th>Activity</th>
<th>Points (of 100 total)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participation (In-class or in Microsoft Teams)</td>
<td>20</td>
</tr>
<tr>
<td>The 5 test questions of your choice</td>
<td>50</td>
</tr>
<tr>
<td>The creation of 2 CHUNKlets</td>
<td>30</td>
</tr>
</tbody>
</table>

In class participation (20 points):
I have a very interactive teaching style that expects everyone to participate in classroom or chat threaded discussions in Teams. Asynchronous learners are encouraged to be engaged in these same chat discussions in Teams as well. The in class conversations allow you to modify and improve your existing perceptions about the topics we’re exploring. Outside of the classroom opportunities are available for engagement through the designated topics channels in Microsoft Teams.

Assessments of learning: Participation is measured by evidence of class preparation through the interactions during class or responses in the chat threaded discussions in Teams (asking relevant questions, showing the ability to express critical thinking). These allow me to establish whether you are actively engaged vs. checking out mentally during class; as well as to assess the asynchronous learners’ comprehension.

CHUNKLet creation and presentation (15 points each, must create 2):
First, review the CHUNK Learning introduction video and further read the information wiki. Then, watch the introductions for setting up and profile and introduction to exploring the Network of Knowledge on CHUNK Learning. Once you are familiar with CHUNK Learning, you are now ready to think about creating CHUNKlets: consider two topics in graph theory for which you would like to create a CHUNKlet either as a "Why" or "How". Create and present two CHUNKlets (presentation can be a report document, narrated presentation slides, demo, video, or other creative ideas). Each student must turn in 2 CHUNKlets that should be 3-10 minutes in length. If you are working in teams, one CHUNKlet counts for a single member of the team (i.e. a 2-member team needs to turn in 4 CHUNKlets). A CHUNKlet can cover more than one topic.

Assessments of learning: The assessment will be driven by the accuracy and clarity of the created content. The following should shape the presentations as well: helpful visual aids, clear, complete and organized presentation (labeled pictures/tables). Table 1 summarizes assessment scoring.

Weekly tests (10 points each, must create 5):
There are 8 take-home tests already posted in our Microsoft Teams site, due in Teams each Friday and can be turned in any time before each test’s due date. While you can practice each of all 8 tests over the concepts learned that week, please turn in exactly 5 tests for grading. I post solutions in Overleaf right after due date for your references, and I strongly encourage you to come ask questions in comparing your test to the solution of the test (keep in mind that there is more than one way to get to a solution). I provide written feedback to each learner in Teams upon returning the graded test the following day. The purpose of weekly take home tests is to facilitate knowledge integration by combining the concepts learned up to that week.

For the Network Profile Summary slide choose a network that interests you, such as your social network from existing social media or create your own (classroom time will be set aside for creating networks). You will be analyzing that network each week, using the concepts learned that week. Consequently, each week you have a choice of turning in one PPT slide summarizing the network’s analysis focused on that week’s topics, yet connected to previous topics as they are relevant. For your network, you may
use your thesis data, may collect/create data, find an existing network/data, or download a network from [http://faculty.nps.edu/rgera/MA4404/NetworkProfileSummaryResources.html](http://faculty.nps.edu/rgera/MA4404/NetworkProfileSummaryResources.html). This test type transitions from knowledge exposure to practice (without mimicking the instructor). During and outside the class you will have the opportunity to experiment on your network, exploring the variety of topics as they are presented and as your intellectual curiosity inspires you. An example of narrated PPT slides can be found here: [first slide](first slide) [third slide](third slide) and [fourth slide](fourth slide) created by Maj Andrew Dietz.

**Assessment Rubric for Weekly Tests:** The grading of the proof assessment is driven by the correctness of the proof, complemented by the clarity and exposition of the writing. The grading of the Network Profile Summary PPT slide uses the knowledge, skill and ability assessment of Table 1 encouraging integration and connection to previous weeks’ concepts.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Task</th>
<th>Detailed Step</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Content</strong></td>
<td>Perform analysis</td>
<td>Correct analysis synthesizing learned concepts</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>A (9-10 points)</strong> relevant &amp; clearly explained findings; clear interpretation of metrics; insightful contextualization of findings; thoughtful synthesis</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>B (8 points)</strong> Minor errors</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>C (7 points)</strong> Significant errors</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>D (6 points)</strong> Major conceptual errors</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>F (0-5 points)</strong> Little to no work of merit</td>
</tr>
<tr>
<td><strong>Presentation</strong></td>
<td>Display results</td>
<td><strong>Clarity and style of graphics</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>A (9-10 points)</strong> clear &amp; succinct slides; correct spelling and mathematical notation; figures and tables are labeled and have captions; consistent in tense and active voice; references are provided conveying the information; clear verbal explanation; correct use of terminology</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>B (8 points)</strong> Minor errors</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>C (7 points)</strong> Significant errors</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>D (6 points)</strong> Incoherent presentation</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>F (0-5 points)</strong> Little to no work of merit</td>
</tr>
</tbody>
</table>

Table 1: Assessment rubric for slides and presentations

**Academic integrity** The honor code can be found in the [NPS student handbook](http://www.nps.edu). If you need reasonable accommodations for a scheduled absence, please see me early in the quarter.